The Market for "Melons":
Quantity Uncertainty and the Market Mechanism
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ABSTRACT

Markets as diverse as labor, healthcare, restaurants, transportation, mobile telephony, and broadband Internet services often have providers that offer flat-rate pricing; usage-based pricing; or both. Using agent-based simulation and analysis of an idealized model of a duopoly with one flat-rate and one usage-based provider, we demonstrate that flat-rate plans are unsustainable in a perfectly competitive market with independent, decentralized decision-making by active, self-selecting, rational utility maximizers engaged in a stochastic, multi-step decision process driving iterative price adjustment. In this duopoly, all customers except those with maximum usage either sequentially or simultaneously defect to usage-based plans, with the remaining heaviest users having equal surplus under either plan. In the absence of usage-based alternatives, a type of market failure can occur. The proximate cause is consumption quantity dispersion. Information asymmetry may also play a role, but in distinction to Nobel Laureate Dr. George Akerlof's *quality* uncertainty in a "Market for 'Lemons'", where the *seller* is advantaged by asymmetric information regarding the *quality* of the product or service being sold, in what we'll call the "Market for 'Melons'" it is the *buyer* that may be advantaged by asymmetric information regarding the ex-ante *quantity* of planned consumption. This asymmetric information is self-defeating, however, since as it eradicates the viability of flat-rate pricing, so it does thereby its own value. Moreover, we argue that effects such as adverse selection and moral hazard have less to do with quality uncertainty, information asymmetry, or morality, than with rational choice by consumers with dispersed consumption under flat-rate pricing: when a provider offers a flat-rate plan, marginal consumption has zero marginal cost to the consumer, who thereby may be rationally indifferent to level of consumption. Other flat-rate market models, such as a monopolist, whose individual customers, rather than defecting, decide to increase their consumption to be equal to or greater than the average, also fail to be sustainable or differentiated. Either all usage evolves in the limit to equal, maximal consumption, with equivalent payments as under pay-per-use; an unbounded spiral of escalating consumption occurs; or insufficient capacity degrades the customer experience. In practice, many factors ranging from transaction costs to behavioral economics and cognitive biases impact the application of these results.

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[1] The author is employed by a Fortune 10 company; however the views expressed herein are his own.

1. INTRODUCTION

Flat-rate pricing schemes have advantages such as simplicity[i] in execution, ease of customer understanding, and alignment with human behavioral anomalies such as loss aversion. However, they also have a number of issues, such as incenting moral hazard, inefficient resource allocation, and unfairness due to subsidization of heavy users by light ones. These issues are more than theoretical, e.g., widely dispersed distributions of wire-line and wireless broadband usage are causing telecommunications service providers such as cable companies[ii] and wireless providers[iii] to rethink flat-rate plans in favor of pay-per-use or tiering, with a majority of telecom executives expecting pricing changes in the next three years[iv]. Confusing things further, in the emerging cloud computing market[v], usage-based pricing is promoted as having benefits over flat rates[vi], whereas in other markets, such as priority mail delivery[vii], it is flat-rate pricing that is promoted as advantageous.

The choice of flat rate vs. usage-based plans may also be framed in terms of an ownership vs. rental decision: for example, owning a DVD vs. incurring a pay-per-view charge or buying a software package or computer vs. using "cloud-based" software or infrastructure service offerings[viii]. It may also be framed as a subscription vs. per instance choice, e.g., subscribing to a newspaper or magazine vs. buying single copies, or even single articles.

There are numerous factors influencing which model might predominate, including consumer behavioral economics effects such as cognitive biases and bounded rationality, transaction costs, existence and availability of information, search costs, and market structure and dynamics.

Almost exactly forty years ago, one of the most cited papers in economics, "The Market for "Lemons": Quality Uncertainty and the Market Mechanism,"[ix] by George Akerlof, was published in the *Quarterly Journal of Economics* (hence the derivative title of this paper). In 2001, together with Michael Spence and Joseph Stiglitz, Dr. Akerlof was awarded the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel—often referred to as the Nobel Prize in Economic Sciences—for insights described therein regarding issues up to and including market failure, due to *information asymmetry*, i.e., when one party in a commercial transaction has information that another doesn't. Akerlof observed that asymmetries exist in a "Market for Lemons," that is, markets for goods such as used cars which may be of uncertain quality. Specifically, only the seller knows whether her car arrived in perfect condition from the dealership and has only been driven carefully on Sundays to church under blue skies, or is a "lemon" due to manufacturing defects, crashes, and/or poor maintenance and repairs. Therefore, buyers, without access to such "hidden" information, are unable to effectively establish a correct price to pay. They won't pay a price that assumes that the car has been pampered, because there is some non-zero probability that it hasn't, lowering the expected value. However, sellers of quality cars won't

conclude transactions at a price representing the expected value, because they would lose money. Such information asymmetries have the perverse effect of destroying the functioning of a market: rather than both good cars and lemons being traded as such, good cars will not be sold as fair value won't be received, and only lemons will trade hands. Akerlof grouped all such effects under "quality uncertainty," that is, uncertainty regarding whether products or services reliably met their stated function, using terms such as "goods of many grades," "good cars and bad cars," and "bad wares and good wares," and suggesting that institutions such as guarantees, brand names, and licensing may ameliorate such quality uncertainty.

However, under the rubric of quality uncertainty, Akerlof includes the case of health insurance, observing that it may be difficult for older individuals to buy medical insurance, calling it "strictly analogous" to the case of selling automobiles. I propose here that it may be *analogous*, but not "strictly" so. More than a matter of semantics, it is dramatically different in key respects, and emblematic of a different mechanism with different behavior: rather than the *seller* uniquely possessing information regarding the *quality* of the good to be sold, it is the *buyer* that uniquely possesses information regarding the *quantity* of planned *consumption* under a flat-rate plan, and buyers have dispersed consumption levels, making a flat-rate plan more attractive to a rational consumer with heavier-than-average consumption, and less attractive to one with lower-than-average consumption.

We will characterize this type of market—which we will refer to, in deference to Akerlof's landmark paper, as a *Market for Melons*—where it is the *buyer* that asymmetrically has information that the seller does not. Such markets also have unstable system dynamics due to the unsustainability of the flat-rate model, as we shall show, given some basic assumptions including a state transition model for price iteration, or *tâtonnement*. These markets include not only "All-You-Can-Eat" buffets—hence the term 'Melons'—but any competitive market where the consumption of a product or service is variable but the total charge is fixed. We can think of the two markets—melons and lemons—as mirror-images or duals of each other. Table 1 provides a comparison, with salient differences in <u>*underlined italics*</u>.

This duality may be generalized away from the specifics of quality uncertainty or quantity uncertainty to a rule addressing goods with heterogeneous value:

> *When value is dispersed, no rational agent will choose to transact an exchange at average value if such an exchange would lead to a loss.*

Specifically, no rational agent would accept a payment of average value for a good, i.e., product or service, known to be of higher-than-average value, nor offer a payment of expected value for a good known to be of lower-than-average value.

	Market for "Lemons"	Market for "Melons"
Dispersed Characteristic	Product *Quality*	Consumption *Quantity*
Advantaged Party	*Seller*, Who Has Asymmetric Information Concerning Ex-Ante *Product Quality*	*Buyer*, Who Has Asymmetric Information Concerning Ex-Ante *Consumption Quantity*
Disadvantaged Party	*Buyer*	*Seller*
Disadvantaged Party's Constraint	Willing to *Buy* Only at a Price *Less Than or Equal To* Expected Value	Willing to *Sell* Only at a Price *Greater Than Or Equal To* Expected Value
Counterparty	*Seller*	*Buyer*
Counterparty's Willingness to Transact	Willing to *Sell* Only at a Price *Greater Than or Equal To* Actual *Quality* Level	Willing to *Buy* Only at a Price *Less Than or Equal to* Actual *Consumption* Level
Market Issue	Transactions Won't Occur When a Rational Offered *Purchase* Price Based On Expected Value of *Product Quality* Is *Less* Than a Rational Offered *Sale* Price Based on Actual *Product Quality*	Transactions Won't Occur When a Rational Offered *Sale* Price Based on Expected Value of *Consumption Quantity* is *More* Than a Rational Offered *Purchase* Price Based on Actual *Consumption Quantity*
Iterative System Dynamic	*Sellers* of *Higher Quality Goods* Exit the Market, *Lowering* the Expected *Quality* of Remaining Transactions	*Buyers* with *Lower Quantity* of *Consumption* Exit the Market, *Raising* the Expected *Consumption Quantity* of Remaining Transactions
Problematic Outcome	Only The *Lowest Product Quality Sellers* Will Remain in the Market	Only the *Highest Consumption Quantity Buyers* Will Remain in the Market
Colloquial Summary	"The bad drives out the good"	"Heavy users drive out light ones"
Problem Resolution	Market Adoption of Pricing Based on *Actual Product Quality*, Rather than Expected	Market Adoption of Pricing Based on *Actual Consumption Quantity* ("Usage-Based"), Not Expected ("Flat Rate")
Example Techniques to Enable Resolution	*Quality* Measurement Processes and Systems, *Seller* Certifications, *Seller* Screening, *Quality* Signaling, etc.	*Quantity* Measurement Processes and Systems, *Buyer* Certifications, *Buyer* Screening, *Quantity* Signaling, etc.

TABLE 1: The Market for "Lemons" vs. The Market for "Melons"

Value dispersion or heterogeneity is key, because if all goods are essentially identical, the minimum equals the expectation equals the maximum. Such dispersion may be based on quantity, quality, or perhaps some other characteristic. The first case can be the classic lemons market, but it could also be a case of quantity uncertainty, where the seller is delivering goods of known quality but uncertain quantity. Clearly, it can also be the case of paying a flat rate for lower-than-average quantity, but might also be the case of paying a flat rate for lower-than-average quality.

There is an interesting interplay between dispersion, asymmetry, and screening. If all customers had identical consumption, a flat-rate plan could be effective, and would trivially equate to a usage-based plan. If a provider must accept all customers, but a customer can choose which type of provider to patronize, then lighter consumers will rationally defect from flat-rate plans, whereas heavier users will rationally remain with those plans. Unsustainability of flat-rate plans is then not a function of information asymmetry, merely of dispersed consumption levels coupled with rational utility maximization. If a provider can screen which customers to serve, but only a customer knows which type of consumer he is, then information asymmetry can play a role. In the case of an all-you-can eat buffet, a glutton can enter, claiming to be on a diet. In the case of health insurance, a three-pack a day smoker can claim to be a non-smoker. Shared ignorance and random or variable consumption improve the viability of flat-rate plans: if all customers' consumption levels are stochastic, neither the customer nor the provider has "inside" information, and pricing in line with the expected value of consumption level is rational and sustainable.

Two effects have been argued to occur in lemons markets, although they actually are present in melons markets: *moral hazard* and *adverse selection*. However, I would argue that these effects are not due to "quality uncertainty" by the buyer, but rather the collision of consumption *quantity* dispersion with flat-rate plans, potentially exacerbated by information asymmetry, and thus characterize melons markets.

Some markets and pricing plans where information asymmetries exist have been described as experiencing an adverse selection problem. For example, health insurance and life insurance are examples of markets where an individual's knowledge of his condition provides the basis for determining how much insurance to buy. Healthy individuals may be predisposed to buy little or no insurance, and those with pre-existing conditions, that is, asymmetric information regarding their future claims, buy a lot. In an extreme case, more than one movie has as its main plot element a murderer who takes out a life insurance policy on his or her victim. In the health insurance context, we then must differentiate between chronic conditions or even genetic or lifestyle predisposition, where there is likely to be at least some validity of expectations

regarding future consumption, vs. acute events such as, say, being struck by an asteroid, which may follow aggregate population statistics, but for which an individual's forecast may be no better than the expectation of a stochastic variable.

It is difficult to see how there is quality uncertainty in such situations, after all, it is not the insurer's competence, capabilities or reliability in either (directly or indirectly) treating a condition, selecting competent healthcare providers to be in-network, or timeliness in paying claims. Instead, the adverse selection "problem" is clearly due to a rational choice of each agent given pricing and expected consumption. Arguably, even the information asymmetry argument is of limited validity: both the customer and the provider may have complete information regarding future consumption, but if this consumption is lighter than average then the consumer surplus is diminished whereas if it is heavier than average then the producer surplus is. Consequently, it is not information asymmetry that is driving adverse selection, but the stark fact that a dispersed distribution will have some values below the mean and some values above the mean, and a flat-rate price determined by mean consumption in a perfectly competitive market of rational agents must represent underpayment by some customers and overpayment by others.

Moral hazard occurs when, e.g., a homeowner who has fire insurance is careless about smoking, lit candles, and furnace maintenance. After all, should the house burn down, the insurance company will pay for it. One can see that "an insurance policy may change the behavior of the insured in a way which makes the event covered by the insurance policy more likely to happen" and the insurer may lack information regarding "whether it was due to an exogenous event or to negligence."[x] Such behavior is not limited to customers of private insurers: it has been observed that businesses such as banks and insurers such as the government can create the same problem: flat-rate Federal Deposit insurance can potentially create risk-seeking behavior among banks.[xi]
Again, while there may be information asymmetries, it would appear difficult to determine where exactly the quality uncertainty lies. Rather, the mechanism arguably is structured to drive emergent behaviors.

In health insurance, two related phenomena occur. In one, moral hazard leads to carelessness regarding one's health. In the other, insured parties will choose greater (and presumably more costly) medical care rather than lesser. A typical view on moral hazard appears to be[xii] that it is due to the intersection of principal-agent models and incentive compatibility, i.e., for a principal such as an insurance company to provide aligned behavioral incentives to agents such as insured parties; "hidden action," or unobservable behavior, e.g., falling asleep while smoking, "hidden information," e.g., propensity to do so, risk, i.e., stochastic processes, normative human behavior, e.g., risk-aversion and expected utility maximization; and the like. Stiglitz asserts that adverse selection is a case of hidden information, whereas moral hazard is a case of hidden action, also a type

of imperfect information, e.g., not being able to observe actions such as whether the insured party fell asleep smoking or it was a faulty heating unit that caught fire.[xiii]

However, as early as 1968, Mark V. Pauly observed that insurers view such behavior, of "demanding more at a zero price than at a positive one…as a moral or ethical problem….together with outright fraud. [However,] the response of seeking more medical care [when insured] is a result not of moral perfidy, but of rational economic behavior."[xiv] Nobelist Sir James Alexander Mirrlees viewed this position as "startling,"[xv] but later goes on to paraphrase John Flemming, agreeing that "it is odd that the problem of self-interested unobservable behavior has come to be called 'moral hazard.'" We can abstract this problem away from morality and unobservable behavior, and show that it is, in line with Pauly, a natural outcome of system dynamics for customers with heterogeneous demand levels in pricing schemes where the marginal price for additional units of consumption is zero. Simply put, if it's (marginally) free, why *not* sample each dessert at the all-you-can-eat buffet. Consequently, instead of "moral hazard," one might just as well describe this using a less succinct but also less loaded phrase: "rational indifference to level of consumption at zero marginal cost."

Complicating things further, emergent effects may not result from information asymmetries that exist where the seller has information that the buyer does not, and where *quality* is the key driver of such asymmetries[xvi]. Other mechanisms may be at work, for example Izquierdo *et al*[xvii], use an agent-based simulation to demonstrate that market failure can occur in the absence of information asymmetries due merely to *quality variability*. *Quantity uncertainty* has also been explored, but typically in the context of uncertainty regarding the quantity of goods to be delivered by the seller to the buyer, not uncertainty regarding the quantity of goods to be consumed under a flat-rate plan.

Pay-per-use service models for computing have arisen recently, such as "cloud" or "utility" computing, where a consumer or enterprise might, for example, "rent" a number of virtual servers or gigabytes of storage for an hour or two. While economists may view pay-per-use as an alternative tariff to flat rates, an enterprise decision-maker attempting to maximize her consumer surplus (i.e., net benefit) might also consider an operating lease on equipment in a leased or owned data center or co-location space at a hosting provider to represent a flat-rate choice for infrastructure vs. a consumption-based cloud computing infrastructure-as-a-service offer. Demand varying intertemporally would then cause potentially dramatic periods of underutilization of fixed resources[xviii], also driving customers to pay-per-use price plans[xix]. And, therefore, it isn't that much of a stretch to view depreciation and cost of capital for an expenditure for equipment and data center as substantially equivalent to a lease which is substantially equivalent to a flat-rate commitment, and view any of these choices against a pay-per-use option.

Of course, pricing is just one element of a complex set of issues that relate to digital information services such as content, Internet access, cellular telephony services, and cloud computing. Numerous other questions arise, such as intelligent agents, search costs, and standards.[xx]

Even the market for primary and secondary education exhibits this bifurcation. Voters, who may or may not be parents and thus customers of primary or secondary education, can vote for an all-inclusive plan, providing money for education via a public school budget regardless of whether they are parents or the number of children they have, or vote down use of tax money for education, thus requiring pay-per-use (private schooling with per-student tuition).

Often, a provider may offer plans of both types to the same customer segments at the same time. For example, restaurants may offer the ability to order "off the menu," or partake of their lavish "all-you-can-eat" buffet. Even individual items may be offered in diverse ways, e.g., "pay-per-use" pricing for omelets, but washed down with a "bottomless cup of coffee" or "unlimited refills." Cellular voice and data plans are often offered in both ways as well, e.g., an "unlimited" plan vs. a basic plan with, say, "35 cents per minute" over the basic usage. In the market for labor, where workers are the vendors and employers are the buyers, both consumption-based (hourly wages) and flat-rate (salaried) plans are available.

There are a wide variety of pricing models in use in various industries, but one choice repeatedly faced is the choice between flat rates and usage-based pricing. Flat rates might appear to have the upper hand: as Le Blanc points out, "flat rate's success appears to be linked to a strong consumer preference for simplicity and previsibility. The flat rate actually helps reduce information asymmetry (fear of being cheated by the access service provider), saves mental transaction costs, and provides a guarantee against a sudden and uncontrolled rise in consumption."[xxi] Simplicity can be important: at one point, a proposal for a non-flat-rate charging scheme for UK roads involved 75 different possible charges, calling into question the ability of customers to understand, much less make optimal decisions, based on rates.[xxii] Yet, the flat-rate model has also been criticized, in that it allegedly: "(1) encourages waste and increases cost, (2) forces light users to subsidize heavy users, and (3) can introduce differentiated service quality only by inefficient segmentation in quality tiers."[xxiii] Flat rates are as often a minority of provider price plans in some industries as they are prevalent in others, e.g., all-you-can-eat restaurants hardly dominate the food services industry. In some industries such as cellular communications, there appears to be a pendulum swing between the models.

Consumer behavior in such environments often exhibits behavioral economic and cognitive biases. For example, Lambrecht and Skiera explore the "flat-rate bias," where customers select flat-rate plans even though a usage-based plan would be less costly given their consumption patterns, and the less common

reverse bias, the "pay-per-use bias," where they would be better off with a flat-rate plan.[xxiv]

As Nahata, *et al*, point out, "the most striking feature of [flat-rate] pricing is that, in spite of the fact that the seller's cost depends on the quantity, the price (or the entry fee) charged to a consumer is independent of the quantity consumed."[xxv] They go on to point out that this is a type of price discrimination, since different customers with different consumption levels end up paying different prices per unit quantity. One may observe that if the customers do not *ex ante* know their future consumption levels, then flat rates do not enable discrimination. However, if users whose consumption is light on a regular basis are charged the same as ones whose consumption is typically heavy, then they do.

In this paper, we prove that assuming rational behavior of customers, a duopoly comprising a flat-rate provider and a pay-per-use provider evolves through customer defection to a terminal equilibrium state where the flat-rate model is essentially valueless. Specifically, if there is a flat-rate price for a service based on average (expected) consumption at a fair (perfectly competitive) price, heavy users of the flat-rate service will underpay, whereas light users, who can pay less via consumption-based plans defect to "pay-per-use". Simultaneously, heavy users will defect from "pay-per-use" to flat-rate plans. As light users defect *from* the flat-rate plans and heavy users defect *to* them, average consumption increases, causing a series of price increases for the remaining flat-rate plan users. A new set of users now overpays, causing them to defect, and the average to increase. Eventually, the system reaches a stable terminal state, but in this state the heaviest users pay just as much whether they are on "all-you-can-eat" or "consumption-based" plans, and all other users have migrated to pay-per-use plans, where their cost is lower than it would be under the "unlimited" plan. Oddly, during and after all of these defections, as long as there is constant re-pricing, there is no change to the total amount spent in aggregate across all customers.

We initially model this market as a duopoly, but it need not be a duopoly in exactly the sense of two providers. We may also consider the market to consist of a monopoly offering both a flat-rate and usage-based plan with no cross-subsidization allowed between plans, or, under conditions of varying demand, as a choice between ownership and rental, where either there is a single dominant rental service market-maker setting prices, or the rental providers all offer similar prices due to perfect or at least a high degree of competition. Such environments are extremely common, after all, the difference between Hertz charging $47.17 per day for a given car and Avis charging $46.95 per day for the same grade of car is not likely to be salient relative to the choice of such a rental vs. buying or leasing that car.

The keys to flat-rate unsustainability are consumption dispersion among customers and the absence of unpredictability of intertemporal variation for a

given customer. If all customers had identical consumption, a flat-rate plan would accurately correlate cost with utility. If a customer had unpredictable consumption across time periods, paying an average price for an expected (average) utility would also be fair and sustainable. Such risk pooling is in effect a minimax strategy: if you don't know whether you will be accident-free or your house will burn down, a maximum payment of a home insurance premium is minimal compared to a potential maximum payment for replacing the entire house. And, if consumption patterns are indeterminate, this is sustainable. A good example is health or life insurance. A broad group of individuals each pays the same premium, unsure who will live to be one hundred, and who will be run over by a truck the next day. However, a pre-existing condition provides an information asymmetry unless disclosed, because current and future consumption of health services is likely to differ from the population mean.

We begin our analysis by considering a simple example, selected for ease of understanding, not for critical relevance to the global economy.

Suppose there are two providers in a very small town: Melons Unlimited (MU, as in mu for mean), which offers all-you-can-eat melon buffets, and, "Slice of Life" (SoL) which lets customers pay by the exact number of slices they consume. This small town only has three melon lovers. Sam Small tends to not eat many melons, say, only one per day.   Max Medium eats three per day, and Larry Large eats five.   Melons cost $.90 each wholesale, and intense price competition has led to a per melon retail price of $1, razor thin margins after SG&A costs.

Sam, Max, and Larry like to go to Melons Unlimited and lunch on melons together. The town is so small that they are MU's only customers. Since Melons Unlimited knows that on average, customers consume 3 melons, MU sets its price at $3.00 for an all-you-can-eat buffet. At some point, Sam Small realizes that the $3.00 each day is more than the $1.00 she would pay by going to SoL. She stops going to lunch with Max and Larry. Max is still fine with going to MU, since he enjoys Larry's company, and doesn't pay any more either way. And, Larry is delighted, because he gets to eat five melons for the price of only three. Now MU only has two customers, which each consume an average of $4.00 worth of melons for a total cost of $8.00 per day, for which MU only receives $6.00 in revenue.

After convening an emergency board meeting, MU decides to raise its buffet price to $4.00, reflecting the average customer consumption of four melons per day. At this point, Larry still benefits, but Max realizes that hanging out with Larry is costing him an extra dollar per day, which adds up to hundreds of dollars each year. He defects to SoL, to pay only $3.00 per day rather than the $4.00.

Fortunately, the board members haven't yet left town on the corporate jet, so they reconvene, realize that the "average" customer (Larry) consumes five melons per day, and raise prices to $5.00. Larry is fine with that price, since he

would have to pay it at either MU or SoL, and so may stick with MU for old times sake, or perhaps switch to SoL so that again he can lunch with his old friends (a network externality).

Now suppose that there are two providers with unlimited plans: MU and Melons Infinity. Assuming that they begin with a similar distribution of customers and therefore similar price points, as soon as MU decides to raise its all-you-can eat price to $4.00, it will see its customers—optimizers that they are—defect to Melons Infinity, where, after all, the melons are just as tasty and the prices are lower.

Of course, this doesn't solve anything, since the average consumption now skews upward at MI, and there is then a low consumption tranche of customers who will defect to SoL, in turn causing MI to either eventually become bankrupt or raise prices as well. We will formalize this inevitable market ecosystem evolution later.

2. PRELIMINARIES

2.1. OVERVIEW OF PRICING

Pricing is a complex art and science at the intersection of economics, strategy, mathematical optimization, human behavior[xxvi], and public policy. Arbués[xxvii] *et al* list market efficiency, equity (fairness), public health, environmental efficiency, financial stability, simplicity, public acceptability, and transparency as among the objectives and constraints of a pricing scheme. Mason[xxviii] observes that firms selecting from among price plan options—not just setting prices—must so decide in light of competitive strategies, positing that two-part tariffs may cause more intense competition than flat rates.

Terms such as price, tariff, and charge often have multiple meanings. For example, the term "tariff" is often used to describe a type of import tax or duty. However, in this context, it is used in the sense of regulatory economics as a specific formula and/or documentation of that formula for specifying a price to charge. Often, price is used in the sense of unit price, with charge then being the total amount of the bill, based possibly on quantity, and adjusted for discounts or other factors.

Flat-rate and usage-based charges go by many other names, in the economics literature, in common practice, and in specific industries or contexts.

Flat-rate pricing is also known as "fixed price," "flat-rate charging," "no-limit-on-quantity [or] buffet pricing[xxix]," "flat-rate tariff," "fixed-rate," "flat," "fixed fee," "fixed entry fee," "block of time" tariffs, "all-you-can-eat," (a.k.a. "AYCE"), "all you can

send"[xxx] (for network bandwidth), "usage-insensitive," "subscription," "unlimited," or "all-inclusive" pricing plans. Examples include some home or mobile broadband plans, some cellular services, car rental services with unlimited mileage plans, all-inclusive resorts, gym memberships, sewer services, health insurance, and deposit insurance. Occasionally, this model is referred to as *prix fixe*. A literal translation of *prix fixe* is fixed price, but in common use, for example, in restaurants, it actually refers to a bundle comprising specific amounts, as opposed to a usage- or volume-insensitive plan.

Usage-based pricing is also known as "usage-sensitive," "linear price," "uniform price," "measured service," "metered," "per-use," "pay-per-use," "usage-sensitive," "usage-fee," "variable-rate," "*à la carte,*" "pay-per-view," "pay-by-the-drink," "pay as you go" (PAYGO), "pay as you drive" (a.k.a. "PAYD"), " "per issue," or "consumption-based" plans. They typically exist for electric utilities, water utilities, natural gas utilities, taxi cabs, and the like. In a pure "usage-based" plan, price is exactly proportional to consumption, e.g., a rate of ten cents per minute, per gallon, per kilowatt, or per mile.

In addition to pure "flat-rate" and "usage-based" plans, numerous other tariffs or pricing plans exist, some of which are illustrated in Figure 1 below.

Non-linear pricing is a broad category that encompasses any charge where the rate is *not* strictly proportional to usage, in other words, virtually all tariffs. Examples include volume discounts, rebates or earned credits such as frequent flyer programs, an otherwise linear price but with a positive y-intercept as with a front-end installation, or access fee, an otherwise linear price but with a negative y-intercept as with a casino's coupon good for $10 in chips, etc. Non-linear pricing is also used specifically to refer to pricing that occurs along a continuous curve, in what perhaps might be better called *curvilinear* pricing.

Multipart tariffs are a type of non-linear pricing, often where different prices are charged for different components of a service, e.g., power generation, transmission, distribution, and/or access.

The *two-part tariff* is the simplest multipart tariff. It has a fixed portion and a consumption-based portion. An example would be traditional telephony plans, with a base rate of, say, $3.95 per month and then ten cents per minute for usage.

Three-part tariffs are another class of multipart tariffs, often with a front-end or fixed monthly portion, a flat rate for usage up to a certain level, and then a surcharge for overage. One example is typical cell phone pricing plans, with an initial cost for the device, a monthly charge for a limited number of minutes per month, and a charge for minutes over the limit.[xxxi] Similarly, car leasing also has three parts, one up-front down payment, a monthly lease rate, and an over-mileage charge. However, three-part tariffs can have different structures, e.g., a

taxi charge normally includes an initial fee regardless of whether the taxi goes anywhere, a portion based on mileage driven, and a portion based on duration of the ride.

Tiered pricing offers a fixed rate for usage within a given range, such as a cell-phone plan where up to 450 minutes of use per month is a flat rate of $39.99, up to 900 minutes of use per month is a flat-rate of $59.99, and so forth. In the Internet Services market, tiers are often associated with particular uplink / downlink bandwidth combinations.

Block or block-rate tariffs provide for a changing series of rates. To illustrate the difference with tiered pricing, a block tariff might specify a price (and therefore incremental charge) of .20 cents per phone call once usage exceeds 3000 minutes per month.

For *block-declining,* or *tapered* tariffs, this rate *decreases* as volume increases.

For *increasing block* or *progressive block* tariffs, the rate increases. While this may seem unusual, it is often followed to ensure basic service such as lifeline telephone rates under Universal Service or sustenance water rates in developing countries.

Dynamic pricing exists when the price for a given product or service varies over time, e.g., airline seats and Vegas hotel rooms, sometimes predictably, or sometimes unpredictably due to the use of yield management algorithms that interact with the forces of market demand.

Peak-Pricing, Off-Peak Discounts and/or Seasonal Tariffs[xxxii] provide one (higher) price for usage during peak periods, and one or more others during lower demand periods. Viewed as fair because peak users are responsible for marginal capex, they can help disincent such usage, and can provide funds for future capacity expansion.

Congestion pricing is a form of dynamic pricing in which prices rise in response to congestion and to proactively prevent it. One everyday example is congestion pricing for driving in the city of London, England[xxxiii], but other interesting proposals exist in other domains, for example, having individual routers in a data network mark packets when the routers are congested, and then charging based on the number of marks[xxxiv], thus incenting users to either avoid using the network or at least to reroute to avoid using paths through congested resources. MacKie-Mason and Varian have proposed "smart markets" for the Internet,[xxxv] where capacity is auctioned off in real-time via a Vickrey Auction, where winners pay the price bid by the highest non-winning bidder, and a variety of other congestion pricing schemes have been proposed.[xxxvi]

Differential Pricing / Price Discrimination occurs when different market segments, i.e., groups of customers, are charged different amounts for the same product. For example, pharmaceutical companies might charge less for the same product in developing countries than in developed ones.

There are a variety of pricing strategies for differentiated products. For example, a first-class seat is priced higher because it provides more legroom, elbow room, and other amenities as well as because it costs more to the airline (square footage of cabin space and lift are the scarce resources on a plane. A resource may not be priced higher just because it costs more, for example, historic Paris Metro subway pricing[xxxvii] provided two prices for the same resource, but a higher price leads to lower demand leads to lower congestion for a "first-class" seat, even though it is the same level of quality.

Priority pricing is familiar to anyone who has used an overnight delivery service, but in addition, is very relevant to the Internet due to the need to potentially allocate scarce resources and due to various traffic types requiring special transmission characteristics, e.g., videoconferencing needs substantial bandwidth, low latency (transmission delay), low jitter (variability of transmission delay) and low packet loss (loss of the data making up the video conference), voice requires little bandwidth by comparison and can tolerate higher packet loss, and email can survive with none of the above. Consequently, various priority pricing schemes have been explored[xxxviii] for use in the Internet, corresponding to the package delivery industry.

Bundling typically provides a discount for several items together—the bundle—versus the price to acquire the individual items on an *à la carte* basis. In *pure bundling*, only bundles are available, in *mixed bundling*, both bundles and individual items are available for purchase. Perhaps counter-intuitively, providing such a discount can actually be a profit maximization strategy when customer segments value different *à la carte* items differently. Typically, a bundle comprises diverse items (sunroof, sport wheels, navigation system, heated seats), but non-linear prices are sometimes considered to be bundles of non-diverse items, i.e., quantity discounts, sometimes called price-quantity bundles, in which all the items are the same good, e.g., French fries or ounces of soda. Typically price-quantity bundles are non-linear: the 20 ounce bag of chips does not cost twice what the ten ounce bag does.[xxxix]

Ramsey Pricing is a regulatory economic notion for markets requiring heavy capital investments in infrastructure, such as global data network services. Ramsey pricing entails pricing at marginal cost, but with lump-sum transfers to recover fixed investments such as network build-outs. Zajac[xl] offers a comprehensive treatment of such pricing, also addressing tradeoffs between Pareto-efficient markets and public objectives such as universal service.
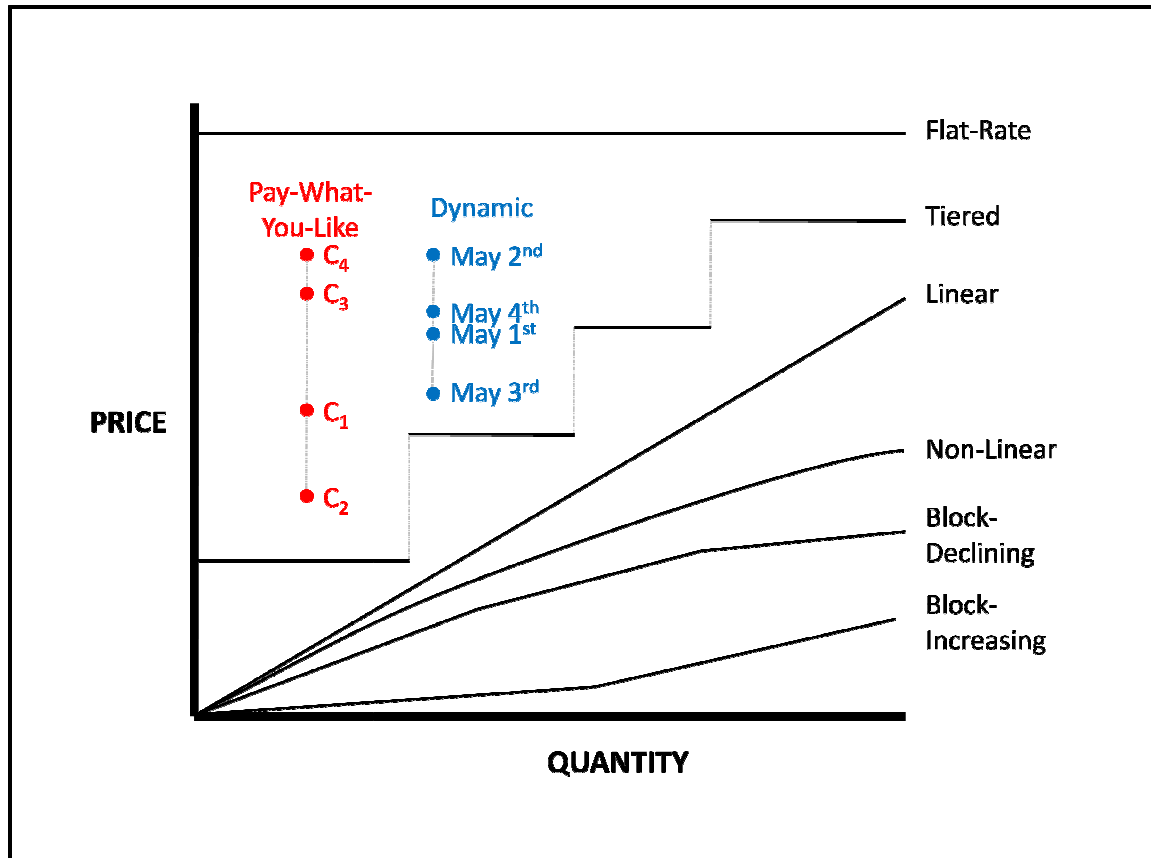
FIGURE 1: Examples of Pricing Plans

Wright Tariffs,[xli] also known as *load-factor tariffs*, are sometimes used in the electric power industry, particularly in Europe. A non-linear tariff, it meters not just total quantity within a time period, but specifies different prices for, say, low instantaneous usage, medium usage, near-peak, and peak. Consequently, two customers both with, say, 24,000 kilowatt hours of use in a day might be charged differently, because one might use exactly 1,000 every hour, but one might use 3,000 only from 9:00 AM to 5:00 PM and 0 otherwise. This is an interesting tariff, because it is structurally similar to the unique and fair Shapley[xlii] value cost-based pricing scheme, in which a customer's price is calculated based on sharing the incremental cost of a resource only with other customers needing that resource. For example, if only 2 customers out of thousands require capacity to deliver peak power, Wright tariffs can translate such a need into an appropriate charge (monetary, and electrical!).

Solving a somewhat related problem, various approaches to multicast pricing also attempt to align price with accurately attributed cost. Multicasting is something akin to broadcasting, except rather than the same content being sent to all customers simultaneously (whether they tune in or not), multicasting selectively sends that content to specific users who may share some, but not all, of the resources used in the transmission.[xliii] If traditional cable and television are

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good examples of broadcast, live pay-per-view and IPTV when more than one customer is watching a show simultaneously are examples of multicast.

 "Pay what you like"[xliv] or "pay what you want"[xlv] where a customer may pay any price they desire without negotiation, including zero, is yet another scheme. It has been applied to a variety of physical goods, such as restaurant meals, and information goods, such as music downloads and "shareware."

Some or all of these may co-exist within a provider's portfolio of offers. For example, a restaurant might offer *à la carte* usage-based pricing ($2.00 per egg), a *prix fixe* (or *menu degustation*) bundle (American breakfast: 2 eggs any style, toast, bacon, OJ, and coffee for $8.99 as a bundle vs. $14.99 if purchased separately), non-linear pricing (buy 2 cups of coffee and get the third free), and flat-rate (enjoy our sumptuous buffet, $19.99 per person). Finally, the tip is "pay what you like," although social norms tend to impact what one "likes."

Lastly, there are numerous other types of pricing well beyond the scope of this discussion, e.g., attorney contingent fees which are a form of value-based pricing.

Typically, flat-rate plans are offered per a given time period, e.g., for a limited number of hours (buffet lunch, 11AM to 2PM), per day (all inclusive resort), per week (timeshare rental), per month (unlimited cell phone use). However, sometimes flat-rate plans are offered for potentially decades (such as a lifetime subscription to a Personal Video Recording service).

2.2.   ASSUMPTIONS

A variety of researchers such as Kokovin, Nahata and Zhelobodko,[xlvi] Bala and Carr,[xlvii] Sundararajan[xlviii], and Fishburn, Odlyzko, and Siders[xlix] have looked at implications of flat-rate, usage-based, or mixed pricing schemes, coming to different conclusions based on assumptions such as marginal costs, distribution of consumption, consumer biases, transaction costs, etc., so we start with key assumptions underpinning our model. Later, we will relax some of these assumptions, but for now we want to explicitly state them. While the long list of assumptions may appear to limit the validity of the analysis, we are actually merely surfacing many elements that may typically be taken for granted. The assumptions may be roughly divided into customer, provider, and market characteristics.

We will assume the following customer characteristics:

- **Self-awareness of consumption level**: *Assumption*: For customers to determine which plan will be advantageous, they must be aware of their

current level of consumption. *Discussion*: For some items, customers are presumably aware of their consumption level, e.g., gallons of milk. For others, they may have visibility into total charges, e.g., monthly bill. For others, they may have *access* to information, but not necessarily *awareness*, e.g., kilowatt hours per month or gallons of water are shown on a bill, but not necessarily instantly recallable. Customer estimates are notoriously inaccurate, e.g., one study indicates that customers may be off by a factor of three in estimating phone call charges.[i]

- **Knowledge of alternative providers of equivalent goods**: *Assumption*: For customers to choose to defect to another provider, they must know which other providers offer the same good. *Discussion*: Bounded rationality does not permit and satisficing does not require complete knowledge of all possible providers / competitors. However, knowing alternate providers subject to isomorphic pricing plans provides a foundation for choice.

- **No search costs:** *Assumption*: Knowledge of these alternate providers and their plans does not entail customer search or information costs. *Discussion*: If search and information costs are non-zero, customer choice is not just dependent on different charges according to provider and price plan, but also the expected value of finding an alternate provider with a lower cost and the ability to amortize the cost of finding that provider across the savings of a future payment stream. While these costs inarguably exist in the real world, information technologies are reducing the marginal cost of such searches to zero in many cases.

- **Visibility into and awareness of price plans**: *Assumption*: For customers to determine whether to defect, they must be able to translate consumption into prices based on existing price plans of those providers. *Discussion*: One study[ii] showed that only two percent of truckers using the NY-NJ Port Authority toll roads knew that there was an off-peak night-time discount. Even among active selectors of E-ZPass, a substantial fraction (35.9%) were unaware of discounts, fewer still (31.5%) were aware generally of discounts but couldn't identify specifics, and even fewer (27.4%) could identify plan specifics, such as, say, a 10% discount from 8PM to midnight. For customers to decide between a flat-rate and a usage-sensitive plan, they must understand the implications of their consumption pattern on total charge.

- **Behavioral elasticity**: *Assumption*: Knowledge of providers and their price plans means nothing if consumers do not have the innate capacity to respond to pricing signals by defecting to an alternate provider. *Discussion*: Later in the analysis, we will also allow for the ability to alter consumption, a different type of behavioral elasticity.

- **Price over loyalty**: *Assumption*: Customers value low-price more than they do loyalty to a particular provider. *Discussion*: Elasticity is irrelevant to switching if the customer values loyalty over price. However, there is true brand loyalty ("My Dad worked there") and what might be described as "apparent brand loyalty."[lii] Kotler suggests that customers may remain loyal to a brand, i.e., provider, due to any number of factors, including habit, indifference, switching costs, and lack of alternatives. He also points out that price may mask loyalty, e.g., being loyal to a provider may turn out to mean being loyal to the lowest price, which may mean defection when a lower cost provider appears.[liii]

- **Self-selection:** *Assumption*: Customers determine which provider to use. *Discussion*: Since the initial model is a duopoly where one provider offers flat-rate and one provider offers pay-per-use, self-selection of a rate plan turns out to be defection if the customer's chosen rate plan is different than the prior time step. Note that there are cases where the payor may not be the user, e.g., "tween" cell phone plans, or where other parties are involved in the selection process, e.g., governments and employers often select preferred providers or identify a short list of acceptable certified providers.

- **Active**: *Assumption*: Customers faced with the opportunity to defect to save money act on the opportunity. *Discussion*: While it is true that not all customers are active in all cases, empirical results do show a reasonable propensity to act. Customers can act, and often, but not always do. They may act by defecting, or they may act by changing their consumption as incented by pricing plans (discussed later). For example, in 1984, AT&T, upon introducing Reach Out America, provided an optional calling plan with a 15% discount for evening (off-peak) calls, which had an uptake of 68% in the first year, and 84.6% by the third year.[liv] This aligns with empirical studies, such as Train *et al*, who determined that raising rates for usage-based service shifts users to flat-rate and raising rates for flat-rate service shifts users to usage-based.[lv]

- **Limited zone of indifference**: *Assumption*: Customers are willing to switch providers when the savings are "somewhat" compelling. *Discussion*: In the initial model that we analyze, customers will, in effect, defect over an infinitesimal price difference, which is not realistic. Interestingly, though, it turns out that there are counterintuitive implications of the level of price indifference. With large enough indifference, no customer will defect, because the indifference may be larger than the dispersion in consumption and prices. Moderate indifference, however, can actually accelerate convergence to a terminal state, because only "larger" defections tend to occur.

- **Decision priority**: *Assumption*: Such active self-selection and thus defection is a priority. *Discussion*: Relative price differential and income effects can impact the urgency of action.

- **Constant marginal utility**: *Assumption*: In this idealized model, we assume constant marginal utility. *Discussion*: Typically in the real world one finds decreasing marginal utility, for example, $1000 is worth less to a millionaire than a homeless person, a 10th slice of pizza is less satisfying than a first one, etc. We will ignore such effects in this idealized model.

- **Independent, local, decentralized**: *Assumption*: Customers do not cooperate with each other, so each customer decision on consumption level and provider is independently determined. *Discussion*: Often one finds social or other network externalities, boycotts, group actions, cooperative negotiations or other behaviors such as an Axelrod-style evolution of cooperation[lvi]. Some models assume a central principal or Walrasian Auctioneer coordinating the market. These may be ignored in this preliminary analysis.

- **Short-term perspective**: *Assumption*: Customers defect based on *current* pricing data only, not intertemporal optimization. *Discussion*: Humans, of course, have a prefrontal cortex and engage in planning and forecasting behaviors. Providers can communicate pricing directions such as planned future price-downs that can impact current decision making. Moreover, some sub-groups, such as the readers of this article, perhaps, can evaluate hypotheticals, project emergent system states based on current actions, and alter behavior accordingly. Examples would be avoiding consumption of blue-fin tuna sushi due to potential extinction, or riding a bicycle to work due to global warming. As we will see, in our idealized model it is very possible for a customer to defect from one provider to another at one point and then defect back at a future point. In the model, we do not let the customer choose not to defect due to the possibility of a future reversal, although this would not impact the conclusion at all.

- **Selfish users**: *Assumption*: Customers act to maximize their own utility, *Discussion*: Altruistic behavior and choices aligned with identity or values are of course well-known. An example might be healthcare legislation, in which market participants, expressing provider plan preferences through democratically-elected representatives, are "selecting" a particular price plan for health insurance services.

- **Repeat purchasing behavior**: *Assumption*: The customer engages in periodic purchases of the item. *Discussion*: If there is no frequent, repeat purchase of substantially equivalent goods, then comparisons are difficult for the customer as well as an analysis. For example, cellular voice service involves the purchase of a similar good from month to month.

Cellular data service may create greater variability as various different apps are downloaded. Finally, home buying is infrequent, and it may be hard to compare a condominium with a starter home with a home bought during peak-earning years with an empty-nester retirement village.

- **Rational customers with no loss aversion premium**: *Assumption*: Customers are neo-classical von Neumann-Morgenstern[lvii] rational expected utility maximizers. *Discussion*: this assumption is key only in the sense that we are modeling customers who will defect for a better price. We are not concerned with buyer risk aversion or probabilistic payoff. Among the axioms, the one that is necessary but too rigorously theoretical for this paper is *Completeness*, i.e., the charge from flat-rate must be less than, equal to, or greater than pay-per-use; *Transitivity* is inessential until we examine oligopolies rather than the base case of a duopoly. *Independence* and *Continuity* are irrelevant since we are not dealing with risk in the scenarios we examine. As is now widely accepted, consumers, at best, exhibit semi-rational behavior. Rather than making purely expected-utility optimizing choices, they exhibit both "flat-rate" and "pay-per-use" biases, as Lambrecht and Skiera have shown[lviii]. A flat-rate bias exists when a customer selects a flat rate even though selecting a usage-based plan would be less expensive, and a pay-per-use bias is the reverse: a customer selects a usage-sensitive plan when a flat-rate would cost less. Lambrecht and Skiera, examining consumer preferences and relevant research in some depth, suggest four causes for the flat-rate bias. The first is the "insurance effect," where customers want to ensure no surprises on their monthly bill due to loss aversion, a human behavioral economic effect where a dollar gained generates less pleasure than a dollar lost does pain. Studies have shown[lix] that customers are willing to pay a premium for flat-rate plans due to this, but we simplify our analysis by eliminating any such premium. The second is the "taxi meter effect" where the hedonic benefit of an experience is reduced by the realization of the need for payment and an immediate payment requirement as opposed to short-term pleasure of consumption with delayed pain of payment. A third is the "convenience effect" where consumers exhibiting bounded rationality by selecting a flat-rate can avoid the cognitive effort, and search and information costs associated with perfectly rational decision-making. The fourth cause, the "overestimation effect," occurs when a customer overestimates his or her usage and therefore the charges that would occur under a usage-based model, and has an inverse effect— underestimation—that leads to a pay-per-use bias. Such effects are found in everything from communication services to gym memberships.[lx]

- **Constant consumption**: *Assumption:* Customers have constant and therefore predictable periodic consumption of the good. *Discussion:* It is central to the thesis of this paper and the formulation of the first model

analyzed that consumption is fixed. Later on we will examine system dynamics effects where consumption rises or is variable.

- **Usage heterogeneity with dispersed demand**: *Assumption:* demand across all customers is *not* identical. *Discussion:* If it were, then flat-rate and usage-based charges would be the same. Some goods have exactly uniform or substantially uniform demand, e.g., mandatory infant immunizations, exactly one per customer (where the customer is defined as the child, not the parent), and consumer durable appliances, such as refrigerators. Other goods have wide dispersion and/or a long tail, for example, one Internet usage study showed a mean data transfer rate of 2 Gigabytes / month, with a standard deviation of 6 Gigabytes / month, perhaps more intuitively understood as a minimum of zero but a maximum of 120 Gigabytes / month, 60 times the average.[lxi] The exact distribution of demand (uniform, triangle, normal, exponential) will alter the specifics of the model, e.g., number of steps to reach a terminal state), but not the general result.

For providers, we will make the following assumptions:

- **Short-term provider price flexibility**: *Assumption:* The initial model assumes that the price changes every time step, although a second variation explored (simultaneous defection) relaxes it. *Discussion:* There is a further implicit assumption that time steps occur in hours, days, weeks, or months, as opposed to say, centuries or millennia. Such price changes are realistic in many businesses, e.g., airlines may change prices on seats several times each day. The increased adoption of pervasive or ubiquitous computing is likely to increase the frequency of price changes.[lxii] However, a variety of factors, ranging from complex corporate decision-making processes to a desire to limit customer confusion to logistics factors such as price tag or menu reprinting to regulatory constraints regarding tariff filings may act to limit flexibility.

- **Knowledge of average consumption:** *Assumption:* The provider can measure average consumption per customer. *Discussion:* To set prices in our model, the flat-rate provider must know not just the total goods sold and/or revenue, but the average per customer. Simply put, the owner of the restaurant must know how many melons each customer eats to set a per-customer price. This is a reasonable assumption, but note that there must be a mechanism to measure number of customers, e.g., recording the number of diners or amusement park goers, as well as goods sold, e.g., counting the number of crates of melons arriving at the loading dock.

- **Rational providers pursuing sustainable businesses:** *Assumption*: Rational, profit-maximizing firms – unwilling to price below marginal cost and operate at a loss. *Discussion*: Firms may not be able to gain a true

determination of marginal cost, and first best pricing may not enable firms to recover capital investments.

- **Short-term perspective:** *Assumption:* Firms make decisions based on current period information, with an objective to be financially sound in the current period. *Discussion:* On occasion, firms may appear to act irrationally, for example, offering below-cost predatory pricing in the short term, with the objective of driving competitors into bankruptcy and then raising prices. In this model, we assume that prices directly reflect costs in the current period.

- **No cross-subsidies:** *Assumption*: Prices are not subsidized from other divisions or product lines. *Discussion*: In the corporation with a portfolio of businesses, it is recommended practice for "cash cows" to fund growth areas of the business.

- **No fixed or overhead costs**: *Assumption*: A stockless distribution model where firms have no overhead or inventory carrying costs. *Discussion*: typically firms have a mix of capital investments, fixed operating costs and variable costs driving unit cost.

- **Billing accuracy:** *Assumption*: bills rendered by all providers are accurate. *Discussion:* In most industries it has occasionally been known for billing problems and disputes to occur.

- **Meaningful time limits**: "unlimited use" plans often have a limit, due to physical characteristics of the delivery mechanism, cultural norms, human satiety or all of the above. For example, one's consumption at an all-you-can-eat buffet is limited by the quantity of food on display or perhaps on premises, one's ability to eat, and peer pressure / social norms if one were to shovel all the available food at the buffet into a wheelbarrow and roll it to one's table.

For the market as a whole, we will assume the following:

- **No barriers to entry**: *Assumption*: New entrants can join the market ecosystem. *Discussion*: Initially we assume a duopoly, but afterwards we will prove some related results regarding multiple flat-rate and/or multiple usage-based providers. This goes along with the notion of perfectly competitive markets, and ties with the lack of sustainability or rationality in a strategy of predatory pricing to attempt to eliminate a competitor since a new competitor can replace the old one.

- **No positive externalities**: *Assumption*: no benefits outside the system. *Discussion*: A good example of positive externalities is Metcalfe's Law effects where a telephone or social network gains value as additional

participants join, above and beyond the intrinsic value of the good. Although Metcalfe's Law may overestimate value in large, real-world networks,[lxiii] the principle of network or other positive externalities is well understood. In any event, we will ignore externalities for this analysis.

- **No negative externalities**: *Assumption*: no costs outside the system. *Discussion*: Congestion is a good example of a negative externality, whether in a crowded restaurant, where it is hard to hear your tablemates, on a crowded highway, or in a congested data network

- **Standard product or service**: *Assumption*: the good being sold is standard both across customers and over time. *Discussion*: highly custom products such as consulting contracts or architecture, engineering, construction are custom-priced, not flat-rate, although they may be usage-based, e.g., time and materials.

- **Uniform quality**: *Assumption*: quality is homogeneous across the standardized good. *Discussion*: although in the real-world quality varies among products and services due to the stochastic nature of manufacturing processes or deliberate versioning or different quality of service, for the purposes of this paper we will assume that all goods are of uniform quality so as to focus on quantity and tradeoffs between flat-rate and usage-based pricing. And, Akerlof's focus is on variable quality coupled with asymmetric information, or to put it differently, asymmetric uncertainty regarding that quality.

- **Perfectly competitive market with cost-based pricing**: *Assumption*: no supplier surplus, pricing at marginal cost. *Discussion*: If there is "leeway" in the market among providers, then profitability becomes an option, and pricing no longer need reflect average cost. In the analysis, we provide an additional factor to extend generality of results beyond marginal-cost pricing, but results apply as long as provider margins are equivalent, whether zero or not.

- **No switching costs**: *Assumption*: Customers make decisions based merely on the lowest price available. *Discussion*: There often are switching costs, ranging from termination fees to retraining to updating electronic billing information. This ties in to the discussion of indifference above. Assumptions of switching costs merely reduce the potential sequences of customer defections to those where each customer has a substantial enough price differential rationale for defection. Ultimately, the end state is essentially the same, except that rather than only heaviest users remaining on flat-rate plans, some "very heavy" users will as well, where the potential savings from defection won't exceed the switching costs.

- **Independent customers and providers:** *Assumption:* No coalitions, cartels, explicit or implicit collusion, agents, brokers, or cooperatives. *Discussion*: A co-op exercising buying power could drive a provider to sell at a loss rather than lose the volume that the customer represents, in turn causing subsidization. An agency, such as an ad agency buying media for advertisers falls into this category. Rosenberg and Clements remark that repeated periods of re-pricing and "undoubtedly well-intentioned" rules such as advance tariff filings "can have the unintended consequence of facilitating price leadership, signaling, or umbrella pricing."[lxiv] "Signaling" in this context refers not to Spence's quality signaling, but communicating prices to competitors.

- **No resale:** *Assumption*: No resale, and thus no arbitrage, splitting, or combining. *Discussion*: If resale is allowed, then a customer could buy an unbounded quantity under a flat rate and resell it to other customers at a net profit. Of course, those customers could also buy a limitless quantity direct from the provider, but the reseller could buy n times the "limitless" quantity and divide it among n customers. Resale, splitting, and/or combining are often prohibited either due to explicit legal or regulatory provisions, or due to lack of cost-effectiveness, e.g., cost of multiplexers or loss of performance in sharing network connections. Note that this does not affect usage-based pricing, but would impact flat-rate. Monopolists or oligopolists may exclusively lease products to eliminate the possibility of aftermarket resale and thus competing with their customers or resellers. Since, in our model, there is no resale and the market is competitive, there is no opportunity for arbitrage where a customer can buy a good and resell it for a profit. Additionally, there is no splitting, so customers may not split the good into two or more pieces and resell for a profit, or combining, where customers buy two or more units and combine them to create a higher value good.

- **Equilibrium:** the market price per unit melon doesn't shift during the period of analysis, and there is matched supply and demand at that price.

- **Perfect information:** *Assumption*: customer demand is known to providers and provider pricing is known to customers. *Discussion*: Stiglitz[lxv] argued convincingly that markets rarely have perfect information.

2.3.   MARKET, CUSTOMERS, PROVIDERS, DEMAND

Let customers, providers, and the market be structured according to the assumptions above. Providers would like to maximize profit (subject to the limits of perfect competition), and customers would like to maximize consumer surplus (meet their demand at the lowest possible cost to themselves.) At any given

time, one or more customers may switch providers, i.e., "defect," and at any given time, providers may adjust prices.

Assume a three-tier idealized economy for a product or service, say melons, which consists of melon growers, restaurants, and customers. We shall assume that melon growing is perfectly competitive, and has reached equilibrium, where the price of a melon (sold by a grower to a restaurant on a wholesale basis) is m. Restaurants utilize a stockless distribution model, and therefore have no inventory carrying costs, and are also perfectly competitive. At equilibrium, the price of a melon to a customer is M. Without affecting our conclusions, we can assume that $M > m$, i.e., there is some margin, or that $M = m$, i.e., that competition has driven the retail price to marginal cost. Let time proceed in steps: $t = 0$, 1, 2, etc. These steps may occur at fixed time intervals or asynchronously—specifics are not necessary for our preliminary analysis of market ecosystem evolution.

Let there be a finite non-zero population of customers indexed from 1 to n as $C = \{c_1, c_2, \ldots c_n\}$, $n > 0$, with the consumption level (demand, or usage) at each time for each customer defined by $U(c_i)$. We will initially assume that for each customer, the level of consumption is fixed, that is, $U(c_i)$ is not time-varying. Note that this does not accurately characterize a number of markets such as, say, health-care services, where one may be perfectly healthy for years, and then have a massive coronary requiring equally massive consumption of health care services. However, in the first part of the discussion, we will leverage this simplifying assumption, which roughly applies in a number of markets: individuals' consumption of daily calories, smartphone users' consumption of bandwidth via data hungry applications and habits, and the like. Without loss of generality, let us also assume that customers are indexed in monotonically non-decreasing consumption order, such that $i < j \Rightarrow U(c_i) \leq U(c_j)$. We will sometimes refer to c_n as c_{max}, since $\max(U(c_1), U(c_2), \ldots, U(c_n)) = U(c_n)$.

We will assume that there are only two restaurants. For mnemonic purposes, we will call the first one "A" (which offers $\underline{A}$ la carte pricing) and the second one "B" (it is an all-you-can-eat $\underline{B}$uffet). By "*à la carte*," we mean usage-based, and by "buffet," we mean flat-rate. Again, we use this convention to ease comprehension of the proofs. We will use the notation $c_i \in A_t$ to mean that customer c_i is a patron of restaurant A at time t. Or, if c_i is a patron of restaurant B at that time, then we denote this as $c_i \in B_t$. The number of customers in A_t, i.e., the size of the set of patrons of A at time t is $|A_t|$, and the size of the set of patrons at time t of B is $|B_t|$. We will assume that the total number of customers in the system is fixed, but that any customer may switch between A and B over time, that is, $|A_t| + |B_t| = |C| = n$, even if $|A_t| \neq |A_{t+1}|$ and $|B_t| \neq |B_{t+1}|$

2.4. PRICING

Let us denote the price that a customer pays or would pay a provider at a given time by $P_{customer,provider,time}$. Specifically:

A: À la carte pricing: The price P that a customer c_i would be charged (were they to defect to A) or is charged (if they are a patron) by restaurant A at time t is simple and time invariant, namely the number of melons consumed times the retail price of a melon: $P_{i,A,t} = U(c_i) \times M$.

B: All-you-can-eat buffet pricing: The price P that a customer c_i of B at time t would be or is charged is $P_{i,B,t}$. As a flat rate, it behaves differently than prices charged to customers of A, as it is charged to all customers of B based on their expected average consumption, regardless of their actual individual consumption:

$$P_{i,B,t} = \frac{\sum_{i=1}^{n} U(c_i) \mid c_i \in B_t}{|B_t|} \times M$$

In other words, the price P charged to any customer of B at time t is an average "all-you-can-eat" price, determined by the average consumption level of the patrons of restaurant B at that particular time, times the price M of a single serving (which includes margin, if any).

Another way of looking at this is that each customer of B at time t pays a price based not on her individual consumption, but based on the expected consumption a customer of B, i.e., the mean consumption of customers of B at time t. Let us use $\mu(U(B_t))$ as a shorthand representation of the average consumption of a patron of B at time t. That is,

$$\mu(U(B_t)) = \mu(U(c_i) \mid 1 \leq i \leq n,\ c_i \in B_t) = \frac{\sum_{i=1}^{n} U(c_i) \mid c_i \in B_t}{|B_t|}$$

If, say, the patrons of B at time t are customers c_1, c_2, and c_3, and their consumption levels are 5, 10, and 12, that is, $U(c_1) = 5$, $U(c_2) = 10$, $U(c_3) = 12$, then the sum is $U(c_1) + U(c_2) + U(c_3) = 27$, and $\mu(U(B_t)) = 27/|B_t| = 27/3 = 9$. So, all that we are saying here is that the price at time t for patrons of B is the average consumption of units (melons) times the price of a unit (melon):

$$P_{i,B,t} = \mu(U(B_t)) \times M$$

Note that, by the definition, all patrons of B pay the same price at any given time:

$$c_i, c_j \in B_t \rightarrow P_{i,B,t} = P_{j,B,t}$$

2.5.   *INITIAL STATE, DEFECTIONS, SYSTEM DYNAMICS*

Initially, customers c_1 through c_n are randomly assigned to be patrons of A or B. We will show that heavy consumers migrate to the all-you-can-eat plan, light ones migrate to consumption-based plans, and the price of the all-you-can-eat plans rises until a terminal equilibrium state is reached in which all of the non-heaviest users are patrons of the *à la carte* plan and the price that the heaviest users pay is equivalent regardless of whether they are patrons of the all-you-can-eat buffet or *à la carte* plans.

A customer may switch providers, i.e., defect. Such a defection for a customer c_i at time t means that $c_i \in A_t$ but then $c_i \in B_{t+1}$, or the reverse, namely $c_i \in B_t$ and then $c_i \in A_{t+1}$.   Such a defection only occurs if the customer believes he can "get a better deal," that is, his payment, given his level of consumption, would be lower. Consequently, if $P_{i,A,t} < P_{i,B,t}$ the customer may defect from B to A and conversely, if $P_{i,A,t} > P_{i,B,t}$ then the customer may defect from A to B.

One subtlety here is that the customer is comparing current pricing at time t. For provider A it will not change, but a customer may defect from A to B at time t only to discover that at time $t + 1$ the price is now higher than it was and by time $t + 2$ that customer would be better off back with provider A. We will rigorously characterize these effects shortly.

We will define a state transition rule as follows:

> **Sequential Defection Rule:** Let D_t be the set of defection candidates at time t, that is, let $D_t = \{ c_i \in A_t,\ P_{i,B,t} < P_{i,A,t} \} \cup \{ c_i \in B_t,\ P_{i,A,t} < P_{i,B,t} \}$. Select one customer $d \in D_t$ at random and switch their provider, so that either $d \in A_t$ and $d \in B_{t+1}$, or $d \in B_t$ and $d \in A_{t+1}$.

In plain English, D_t is the subset of customers who could get a better deal at time $t,$ and since they are active, rational, surplus maximizers, the sequential defection rule selects <u>one</u> of those customers (at random) and makes them a patron of a different (and cheaper for them) provider at time $t + 1$.

In turn, this drives a *tâtonnement*, or iterative re-pricing process for provider B.

3. PAY-PER-USE DOMINANCE

With those definitions in mind, we now will prove the fundamental result of this paper, which we can call the Pay-Per-Use Dominance Theorem. In game theory, one strategy dominates another when regardless of what strategy the other player(s) select(s), the payoff is higher than an alternate strategy. If we consider price plan selection to be a game whose payoff is measured in expected total customers, where the strategic choice is the selection of pricing strategy, then pay-per-use dominates flat-rate:

> ***Theorem 1 (Pay-Per-Use Dominance):*** Suppose that a finite number of customers $C = \{c_1, c_2, \dots c_n\}$, $n > 0$, with consumption function $U: C \to R^+$ are randomly assigned at time $t = 0$ to two providers: A, offering pay-per-use pricing where $P_{i,A,t} = U(c_i) \times M$, and B, offering flat-rate pricing, where $P_{i,B,t} = \mu(U(B_t))$, such that provider B has at least one customer assigned. Let the h (equally) heaviest users of C be $c_{n-h+1}, \dots c_{n-1}, c_n$ such that $U(c_{n-h+1}) = \dots U(c_{n-1}) = U(c_n)$. If at each step the sequential defection rule is applied, then within a finite number of steps the system will reach a terminal state T where:
>
> 1) $c_1, c_2, \dots c_{n-h} \in A_T$
>
> 2) $n - h + 1 \leq i \leq n \to P_{i,B,T} = P_{i,A,T} = U(c_n) \times M$

In plain English, all except those customers that are the heaviest users end up migrating to the *à la carte* plans, and the heaviest users end up paying the *à la carte* rate whether they use the *à la carte* plan or not. Due to the fact that heaviest users will pay the same amount from either provider, there are actually a number of terminal equilibria, which depend on the initial random distribution of customers to providers as well as the exact sequence of defections. For example, consider the trivial case where all users have identical consumption, and therefore are all "heaviest users." A heavy user of a consumption-based plan will have no incentive to defect, and vice versa. Consequently, the terminal state is the initial state, and dependent on the initial random distribution. There are then $2^n - 1$ terminal states: each customer can be in either plan, leading to the 2^n possible states except for the unallowable initial state where B has no customers (thus the "-1"). Where there are a variety of demand levels, other users may defect before a heavy user decides to, and once all the lower-consumption users have defected to the *à la carte* plan, there will be no incentive for that heavy user to, in which case there are also potentially many terminal states unless there is only one "heaviest" user.

3.1.   LEMMAS

At any given time, a customer may rationally choose to defect from provider A to provider B, or a customer may choose to move from provider B to provider A. Let us consider the conditions under which either might happen, and the net impact.

Proposition 1: A customer c_i can defect from an *à la carte* plan to an all-you-can-eat plan at time t if and only if his usage is heavier than the current all-you-can-eat average, that is, $\mu(U(B_t)) < U(c_i)$.

> **Proof**: According to the sequential defection rule, a customer c_i can defect from A to B at time t, that is, $c_i \in D_t$, if and only if $P_{i,B,t} < P_{i,A,t}$. However, since $P_{i,B,t} = \mu(U(B_t)) \times M$ and $P_{i,A,t} = U(c_i) \times M$, it must be that $P_{i,B,t} = \mu(U(B_t)) \times M < P_{i,A,t} = U(c_i) \times M$, so $\mu(U(B_t)) \times M < U(c_i) \times M$, and therefore, eliminating the M from both sides, $\mu(U(B_t)) < U(c_i)$. ∎

Proposition 2: A customer c_i can defect from an all-you-can-eat plan to an *à la carte* plan at time t if and only if his usage is lower than the current all-you-can-eat average, that is, $U(c_i) < \mu(U(B_t))$.

> **Proof:** A customer c_i can defect from B to A if and only if $P_{i,A,t} < P_{i,B,t}$. However, since $P_{i,B,t} = \mu(U(B_t)) \times M$ and $P_{i,A,t} = U(c_i) \times M$, it must be the case that $P_{i,A,t} = U(c_i) \times M < P_{i,B,t} = \mu(U(B_t)) \times M$. Then $U(c_i) \times M < \mu(U(B_t)) \times M$, and therefore, eliminating the M from both sides, $U(c_i) < \mu(U(B_t))$. ∎

Proposition 3: Let there be at least two customers on the all-you-can-eat plan: $c_i, c_j \in B_t, i \neq j$, with different consumption levels, i.e., $U(c_i) \neq U(c_j)$. Then there is at least one customer $c_i, \ 0 < i \leq n - h,$ that can defect to the *à la carte* plan.

> **Proof:** Let customer $c_{min} \in B_t$ be a customer with the lowest consumption on the all-you-can-eat plan. Because there is at least one customer with a higher consumption, the average $\mu(U(B_t))$ must be higher, that is, $U(c_{min}) < \mu(U(B_t))$ But then, by Proposition 2, c_{min} can defect. ∎

Proposition 4: If all customers of the all-you-can-eat plan have the same usage at time t, none will defect at that time.

> **Proof:** By contradiction. Let a customer that can defect be c_i. Then by Proposition 2, $U(c_i) < \mu\big(U(B_t)\big)$. But if all customers in B_t have the same consumption, then $U(c_i) = \mu\big(U(B_t)\big)$. Since both cannot be true, there is no such customer. ∎

Proposition 5: $|B_0| > 0 \rightarrow |B_t| > 0, \; \forall t > 0$, i.e., if in the initial state there is at least one customer on the all-you-can-eat plan, there will always be at least one customer in the plan.

> **Proof:** Whenever there is exactly one customer in the all-you-can-eat plan, all customers in the plan (trivially) have the same consumption, so by Proposition 4, none will defect. Consequently, there can only be defections into the plan or no defections. If there are defections in, there is at least one customer in the plan. If after defections into the plan, there are defections out of the plan, and at some point the defections out of the plan result in there being only one customer in the plan, we have returned to a state where there can be no defections out of the plan. ∎

Proposition 6 (Monotonically Increasing Prices): $P_{i,B,t} < P_{i,B,t+1}, 0 \leq t < T$, that is, the all-you-can-eat price monotonically increases, regardless of whether a customer defects from all-you-can-eat to *à la carte* or from *à la carte* to all-you-can-eat.

> **Proof**: If the only permissible defections are for users with heavier than B's average consumption to join B or users with lower than B's average consumption to defect from B to A, the average consumption of B must increase. More formally, suppose c_i defects from A at time t to B at time $t + 1$. Then, by Proposition 1, $\mu\big(U(B_t)\big) < U(c_i)$. But then, since adding a value to a set that is above average increases its average, $\mu\big(U(B_{t+1})\big) > \mu\big(U(B_t)\big)$. Conversely, suppose c_i defects from B at time t to A at time $t + 1$. Then, by Proposition 2, $U(c_i) < \mu\big(U(B_t)\big)$. But, since removing a value from a set that is lower than average increases its average, $\mu\big(U(B_{t+1})\big) > \mu\big(U(B_t)\big)$. In either case, the price $P_{i,B,t}$ is based on the average consumption, namely, $P_{i,B,t} = \mu\big(U(B_t)\big) \times M$, and, $P_{i,B,t+1} = \mu\big(U(B_{t+1})\big) \times M$. But, since we just showed that $\mu\big(U(B_t)\big) < \mu\big(U(B_{t+1})\big)$, we know that $\mu\big(U(B_t)\big) \times M < \mu\big(U(B_{t+1})\big) \times M$, thus $P_{i,B,t} < P_{i,B,t+1}$. ∎

In practice, one can imagine finer gradations of timing than we have specified. For example, after re-pricing, individual customers might evaluate the possibility

of defecting, then one makes the decision, then providers are switched, then the new pool of customers is evaluated, then another re-pricing decision is made. Do we need to worry about these micro-timing steps? For example, suppose a customer decides to defect at time t. If the act of defecting changes the relative prices of A and B, so that it was no longer a smart decision and the customer defects right back, there are issues of getting stuck in an infinite loop, as well as potential micro-timing issues: if the customer is considering the switch and the provider incorporates that customer's consumption into the price proposal, some switches might never happen. The next few propositions show that these issues aren't a concern.

Proposition 7: If a customer c_i has defected from the all-you-can-eat plan to the *à la carte* plan, he will never return to the all-you-can-eat plan, that is, $c_i \in B_t$ and $c_i \in A_{t+1} \Rightarrow c_i \in A_{t+1}, c_i \in A_{t+2}, \dots c_i \in A_T$.

> ***Proof:*** If customer $c_i \in B_t$ but defects so that $c_i \in A_{t+1}$ we know from definitions and from Proposition 2 that $P_{i,A,t+1} = P_{i,A,t} < P_{i,B,t}$. By Proposition 6, $P_{i,B,t} < P_{i,B,t+1}, 0 \leq t < T$. But for customer c_i to defect back to B at a later step, say, $t + k, k > 0$, Proposition 1 tells us that it would need to be the case that $P_{i,B,t+k} < P_{i,A,t+1}$. But then $P_{i,B,t+k} < P_{i,A,t+1} < P_{i,B,t}$, contradicting Proposition 6. ∎

Note that the reverse is not true: it turns out a customer can defect from an à la carte plan to an all-you-can-eat plan when the price of the all-you-can-eat plan is lower, but then as the price escalates, decide to defect back. However, the customer can not immediately do so:

Proposition 8 (Decision Stability): If it is a rational decision for a customer c_d to defect at time t and the customer does so, that same customer c_d cannot defect back at time $t + 1$.

> **Proof**: A defection followed by another defection could only occur from A to B to A or from B to A to B. Proposition 7 rules out the latter, so let us consider the former, where $c_d \in A_t, c_d \in B_{t+1}$ and $c_d \in A_{t+2}$. We will show this is not possible by contradiction.
>
> If c_d defects at time t, we know from the sequential defection rule that $c_d \in D_t = \{ c_i \in A_t, P_{i,B,t} < P_{i,A,t} \} \cup \{ c_i \in B_t, P_{i,A,t} < P_{i,B,t} \}$. But since $c_d \in A_t$, we know that $P_{i,B,t} < P_{i,A,t}$.
>
> Then, $\mu(U(B_t)) \times M < P_{i,A,t} = U(c_d) \times M$, so $\mu(U(B_t)) < P_{i,A,t} = U(c_d)$. Let this non-zero positive difference be represented by δ, that is,

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$U(c_d) = \mu\big(U(B_t)\big) + \delta$. But for the customer to defect back at time $t+1$, according to Proposition 2 we would need to have $U(c_d) < \mu\big(U(B_{t+1})\big)$. Then,

$$U(c_d) < \mu\big(U(B_{t+1})\big) = \frac{\big(\mu\big(U(B_t)\big) \times |B_t|\big) + U(c_d)}{|B_{t+1}|}$$

$$= \frac{\big(\mu\big(U(B_t)\big) \times |B_t|\big) + \big(\mu\big(U(B_t)\big) + \delta\big)}{|B_{t+1}|}$$

$$= \frac{\big(\mu\big(U(B_t)\big) \times |B_t|\big) + \big(\mu\big(U(B_t)\big)\big)}{|B_{t+1}|} + \frac{\delta}{|B_{t+1}|}$$

$$= \frac{\mu\big(U(B_t)\big) \times (|B_t| + 1)}{|B_{t+1}|} + \frac{\delta}{|B_{t+1}|}$$

Since there was a defection *into B* at time t, we know that $|B_t| + 1 = |B_{t+1}|$, so:

$$= \mu\big(U(B_t)\big) + \frac{\delta}{|B_{t+1}|}$$

However, we know that $U(c_d) = \mu\big(U(B_t)\big) + \delta$, and by Proposition 5 $|B_t| > 0$, so $|B_{t+1}| = |B_t| + 1 > 1$, and δ is positive, so

$$= \mu\big(U(B_t)\big) + \frac{\delta}{|B_{t+1}|} < \mu\big(U(B_t)\big) + \delta = U(c_d)$$

Therefore, based on our initial inequality that $U(c_d) < \mu\big(U(B_{t+1})\big)$, we would have:

$$U(c_d) < \mu\big(U(B_{t+1})\big) = \mu\big(U(B_t)\big) + \frac{\delta}{|B_{t+1}|} < \mu\big(U(B_t)\big) + \delta = U(c_d)$$

Since $U(c_d)$ can't be less than itself, we have our contradiction. ∎

The exact number of state transitions, that is, the value of T, is dependent on the initial distribution of customers, e.g., allocation to A vs. B, the stochastic process of sequential defections, and the consumption function $U()$. However, we can still show that there aren't any infinite loops of defections, and therefore that the number of state transitions is finite, in fact, less than twice the size of C.

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Proposition 9 (Finite Termination): T is finite and $0 \leq T < 2 \times |C|$.

> **Proof:** A given customer may defect from **A** to **B** and then may defect from **B** to **A**. By Proposition 7, however, at this point no further switch is possible for that customer. Moreover, for at least one customer with the heaviest demand, that customer may defect from **A** to **B**, but then will never defect back from **B** to **A**, so the maximum number of transitions is $2 \times |C|$, less the at least one impossible transition. Consequently, $T < 2 \times |C|$. Moreover, since **C** is finite, T must be finite. Finally, we note that an initial (random) distribution may be terminal, if one or more heaviest users and no other users are $\in B_0$, thus T may equal 0. ∎

It is worth noting that T can be 0, since a (random) initial state where there are one or more heaviest users in B and all other users in A is terminal. However, we can also construct structured, if pathological, defection sequences that don't reach T up until $2 \times (|C| - 1)$. Let's assume that $U(\ )$ is formed such that for c_i, $U(c_i) = i, 1 \leq i \leq n$. Let the initial state have all customers in **A** except for c_1, leading to $P_{i,B,0} = 1$. Let the next defection be c_2, leading to $P_{i,B,1} = \frac{1+2}{2} = 1.5$. Then let the next defection be c_2, leading to $P_{i,B,2} = \frac{1+2+3}{2} = 3$. It is clear that each customer in increasing order will be incented to defect, since the average consumption $\mu(U(B_t))$ is always less. Consequently, there will be $n - 1$ defections from **A** to **B**, until a milestone is reached wherein all customers are in **B**. At this point, and in any order, all customers will defect from **B** to **A** except for c_n, leading to another $n - 1$ defections. Consequently, there are $2 \times (n - 1) = 2 \times (|C| - 1)$ defections in this particular sequence.

Knowing that both extremes are possible is interesting, but what happens in practice? Interestingly, running a variety of simulations on customers with sequential usage, where for each c_i, $U(c_i) = i, 1 \leq i \leq n$, etc., leads to an apparent scale-free invariant that $E(T) \cong .852 \times |C|$, that is, the expected value of the number of steps to terminate is about $.852n$, when the number of customers is n. Roughly speaking (subject to random initial assignment), half of **C** will be initially assigned to **A**, and half will be initially assigned to **B**. The bottom half (in terms of consumption) of **A** will never defect, thus contributing 0 transitions. The bottom half of **B** will always defect once, contributing $.25n$ transitions. The top half of **B** (except the heaviest user c_n if initially in **B**) will always defect once, contributing roughly another $.25n$. The remaining approximately $.352n$ defections come from $.176n$ out of the $.25n$ customers that defect from **A** to **B** and then defect back (excluding again, the heaviest user c_n if initially in **A**), each contributing 2 transitions. This is a non-trivial process to characterize, because the eligibility of these customers to transition changes over time, dropping to zero as the price increases to the maximum, and the probability of an eligible customer being selected also changes due to the evolution of **D**. Different distributions of customer usage lead to different expected values for T, but they will all be within the interval specified by Proposition 9.

Counter-intuitively, even though most customers act to reduce their spend by defecting to a lower cost-provider, the total amount spent in aggregate across all customers doesn't change at all.

Proposition 10 (Spend Invariance): The amount spent in aggregate across all customers of A and B is invariant over time and is:

$$\sum_{i=1}^{n} P_{i,A,t}, c_i \in A_t + \sum_{i=1}^{n} P_{i,B,t}, c_i \in B_t = \sum_{i=1}^{n} U(c_i) \times M$$

Proof: Since $\forall t, A_t \cup B_t = C$ and $A_t \cap B_t = \emptyset$, the total spend across patrons of A and B and time t is the sum of the prices paid by patrons of A plus the sum of the prices paid by patrons of B:

$$\sum_{i=1}^{n} P_{i,A,t}, c_i \in A_t + \sum_{i=1}^{n} P_{i,B,t}, c_i \in B_t$$

Substituting for the definition of $P_{i,A,t}$, we get:

$$= \sum_{i=1}^{n} U(c_i) \times M, c_i \in A_t + \sum_{i=1}^{n} P_{i,B,t}, c_i \in B_t$$

Substituting for the definition of $P_{i,B,t}$, we get:

$$= \sum_{i=1}^{n} U(c_i) \times M, c_i \in A_t + \sum_{i=1}^{n} \mu(U(B_t) \times M, c_i \in B_t$$

Since B_t has $|B_t|$ members, and the price charged to each is identical, this is:

$$= \sum_{i=1}^{n} U(c_i) \times M, c_i \in A_t + |B_t| \times \mu(U(B_t) \times M$$

Then, from the definition of $\mu(U(B_t)$:

$$= \sum_{i=1}^{n} U(c_i) \times M, c_i \in A_t + |B_t| \times \{(\sum_{i=1}^{n} U(c_i), c_i \in B_t) \times {}^{1}/_{|B_t|}\} \times M$$

Cancelling out the B_t, this leaves us with:

$$= \sum_{i=1}^{n} \boldsymbol{U}(c_i) \times M , c_i \in \boldsymbol{A}_t + \{\sum_{i=1}^{n} \boldsymbol{U}(c_i), c_i \in \boldsymbol{B}_t \} \times M$$

The M can be brought within the sum, and since $\forall t, \boldsymbol{A}_t \cup \boldsymbol{B}_t = C$ and $\boldsymbol{A}_t \cap \boldsymbol{B}_t = \emptyset$:

$$= \sum_{i=1}^{n} \boldsymbol{U}(c_i) \times M \quad \blacksquare$$

3.2.   *PROOF OF THEOREM 1*

Now, with the preliminaries as well as some additional insights out of the way, we need to show two things: namely that: (1) $c_1, c_2, c_{n-h} \in \boldsymbol{A}_T$, that is, all the non-heaviest users are on the *à la carte* plan by the terminal state; and (2) that $n - h + 1 \leq i \leq n \rightarrow \boldsymbol{P}_{i,B,t} = \boldsymbol{P}_{i,A,t} = \boldsymbol{U}(c_n) \times M$.

> ***Proof:*** We prove (1) by contradiction. Suppose that we have reached the terminal state and there is an $i, 1 \leq i \leq n - h, c_i \in \boldsymbol{B}_t$. But then $\boldsymbol{U}(c_i) < \boldsymbol{U}(c_n)$. Either $c_n \in \boldsymbol{B}_T$ or $c_n \in \boldsymbol{A}_T$. If $c_n \in \boldsymbol{B}_T$ then there are at least two customers of $\boldsymbol{B}_T$, namely c_i and c_n. But then by Proposition 3 there is at least one customer (namely c_i) that can defect, so the state cannot be terminal.
>
> Conversely, suppose $c_n \in \boldsymbol{A}_T$. Since $c_i \in \boldsymbol{B}_T$, $\boldsymbol{P}_{i,B,T} < \boldsymbol{U}(c_i) \times M$. But $\boldsymbol{P}_{n,A,T} = \boldsymbol{U}(c_n) \times M$. Therefore, by Proposition 1, customer c_n can defect, so the state cannot be terminal.
>
> We now prove (2). Since there was a customer initially in the all-you-can-eat plan, by Proposition 5 there is at least one customer in that plan in the terminal state. As we have shown, there are no non-heavy users in the plan by the terminal state, therefore the one or more customers are all heavy users. Then, by the definition of the price of the plan at time T, $\boldsymbol{P}_{i,B,T} = \mu(\boldsymbol{U}(\boldsymbol{B}_T)) \times M$. But if the price of the plan is based on the average of the usages, and all the usages are heaviest, then $\boldsymbol{P}_{i,B,T} = \mu(\boldsymbol{U}(\boldsymbol{B}_T)) \times M = \boldsymbol{U}(c_n) \times M = \boldsymbol{P}_{i,A,T}$, given $n - h + 1 \leq i \leq n$. $\blacksquare$

4. AGENT-BASED MODEL SIMULATION RESULTS

Agent-based models[lxvi] can be an effective means of illustrating emergent behavior up to the scope of validity of the model. We use a simple simulation to help illustrate the behavior of this stochastic iterative pricing rule. Appendix 1 contains a simple JavaScript simulation packaged as an HTML web page that can run in any JavaScript-enabled web browser.

Here is an example of a simulation on one thousand customers, initially distributed randomly between a flat-rate and usage-based plan. The customers have consumption levels in a non-random quasi-uniformly distributed usage pattern, with each customer having an initial usage commensurate with its index, as before, i.e., c_i, $U(c_i) = i$, $1 \le i \le n$. For simplicity, but without loss of generality, $M = 1$, so $P_{i,B,t} = \mu(U(B_t))$. In a run that we will discuss, the initial state has $|A| = 491$, $|B| = 509$, and thus, of course, $|C| = 1,000$. Initially, based on this random assignment, $P_{i,B,0} = 504.28$. In this run, $T = 838$.

4.1.   PRICE

As can be seen via computer simulation results, the average consumption μ, and thus the price (we let $M = 1$ for the sake of simplicity), monotonically increase at each step until the expected terminal state is reached. Figure 2 shows a chart of price $P_{i,B,t}$:
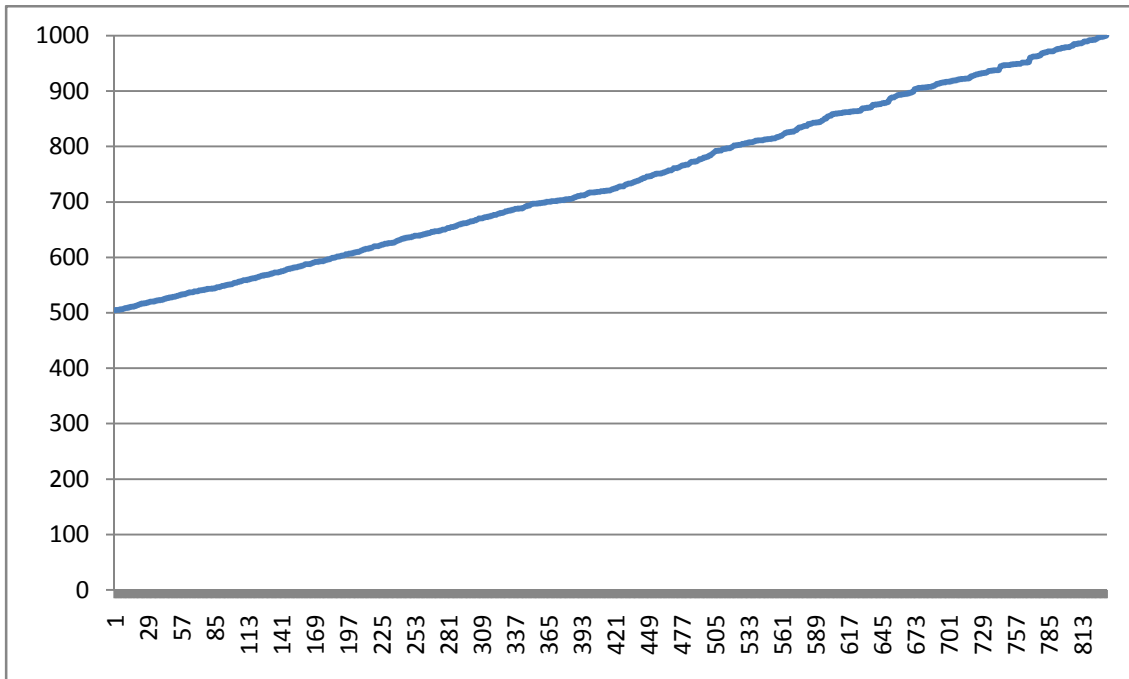

FIGURE 2: Flat-Rate Price Simulation Results on 1,000 Customers

Somewhat surprisingly, the line appears to be straight. We might have guessed that it would be, say, hyperbolic or sigmoid, but the average price increase each step does not appear to vary. Running a simulation on 36,000 customers[2], we can more clearly see *exactly* what is happening in Figure 3:
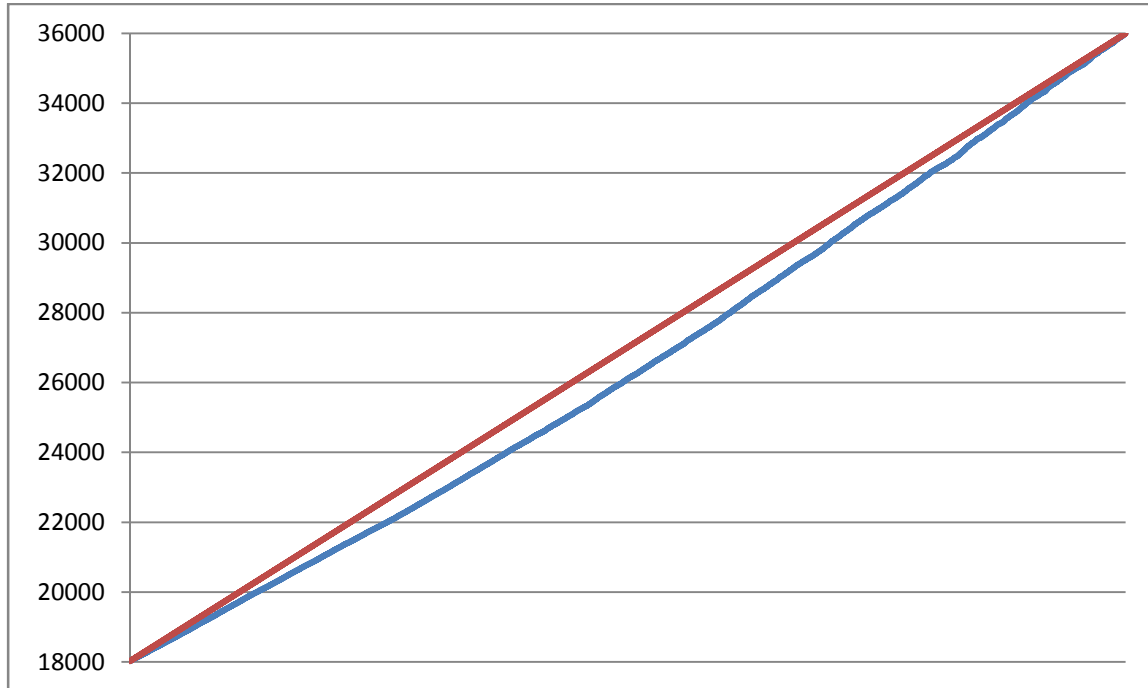

FIGURE 3: Flat-Rate Price Simulation Results on 36,000 Customers

We see that the run actually generates a slightly concave upward curve. The bottom line (in blue) shows the price starting at $18,019.37882$ at $t = 0$, and when $t = T$, which in this case is at step 30,587, we reach the end price of 36,000. The top line (in red) is a straight line between those two points, representing an increment in price at each step of $(36,000 - 18,019.37882) / 30,587 = .58785174$.

While virtually any extreme can happen per Proposition 9, simulation runs appear to show a typical pattern. If the number of customers $|C|$ is n, with consumption levels in the series $1, 2, 3, ... n$, then we can expect that the initial price is $n/2$ (remember, we've let M be 1.) The terminal price, of course, is just n. Here is why the curve behaves the way it does. Let us call the largest possible change in price at time t $\Delta_{t,max}$ and the expected value of the change at time t $E(\Delta_t)$ In the early stages, $E(\Delta_t)$ is small because $|\boldsymbol{B}_0|$ is large and no defection can alter the average very much. For example, we can expect that $|\boldsymbol{B}_0| \cong n/2$, because we expect that about half of $\boldsymbol{C}$ will initially be randomly assigned to $\boldsymbol{B}$ and we expect that $\mu(U(\boldsymbol{B}_0)) \cong (n + 1)/2$ because the indices and thus usage levels run

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[2] The number 36,000 was selected because it leads to termination in less than 32,000 steps, which is the maximum number of data points that Microsoft Excel can currently chart.

from 1 to $n+1$. Consequently, a defection at or near $n/2$ will not change μ significantly. However, a defection of a customer with usage of either 1 or n has a difference from the mean of $n - ((n+1)/2)$, but the impact on μ of the defection must be divided by the size of the set, which will now be $n/2 + 1$ for a defection in or $n/2 - 1$ for a defection out. Because this denominator is so large, the impact is small, in fact, less than unity.

To see this, consider what happens to the average of a set with k members and mean m when another element is added of size $2m$. The initial members follow the obvious identity $m = (k \times m)/k$. When we add the new element, we have a new mean of $\frac{(k \times m) + 2m}{k+1}$, so the change provides us with $\Delta = \frac{(k \times m) + 2m}{k+1} - \frac{(k \times m)}{k}$. Multiplying both sides by $(k+1)k$ gives us $(k+1) \times k \times \Delta = k^2 m + 2km - k^2 m - km$. Cancelling terms, and dividing by $(k+1)k$ leaves us with $\Delta = \frac{m}{k+1}$. If we now plug in $(n+1)/2$ for m and $n/2$ for k, we have $\Delta_{0,max} = \frac{(n+1)/2}{\frac{n}{2}+1}$. Through a similar sequence of operations, we simplify this to $\Delta_{0,max} = \frac{n+1}{n+2}$. In the limit this is unity, i.e., as $n \to \infty$, $\Delta_{0,max} \to 1$, but for any finite n it is just short of 1.0. Of course, since we assumed a quasi-uniform distribution, the defection can lie anywhere on the interval $[1, n]$, and therefore $E(\Delta_0) = \frac{(n+1)}{2(n+2)} \to \frac{1}{2}$.

However, for large t, say between $T-1$ and T, Δ_{max} can be $n - \frac{n+1}{2}$, which will occur if $\boldsymbol{B}_{T-1} = \{c_1, c_n\}$ (where $\boldsymbol{U}(c_1) = 1$ and $\boldsymbol{U}(c_n) = n$) and then $\boldsymbol{B}_T = \{c_n\}$, and thus $\Delta_{T-1,max} = P_{i,\boldsymbol{B},T} - P_{i,\boldsymbol{B},T-1} = n - \frac{n+1}{2} = \frac{n-1}{2}$. The expected values of $E(\Delta_t)$ as $t \to T$ are non-trivial to determine, since they are the last states of a complex Markov process, but clearly are larger than they were in the initial states. Consequently, the price changes are, in terms of maximum possible and in terms of average higher as the *tâtonnement* progresses, leading to increased steepness.

So, we have an average where the numerator to determine the mean $\mu(\boldsymbol{U}(\boldsymbol{B}))$ has gotten larger and the denominator representing $|\boldsymbol{B}|$ gets smaller and smaller, enabling Δ_{max} to get larger "somewhat" hyperbolically. The curve is not exactly hyperbolic since the numerator increases over time, but quite close. Figure 4 is a scatter plot of price increases in a particular simulation run as time progresses, showing exactly the effect described:
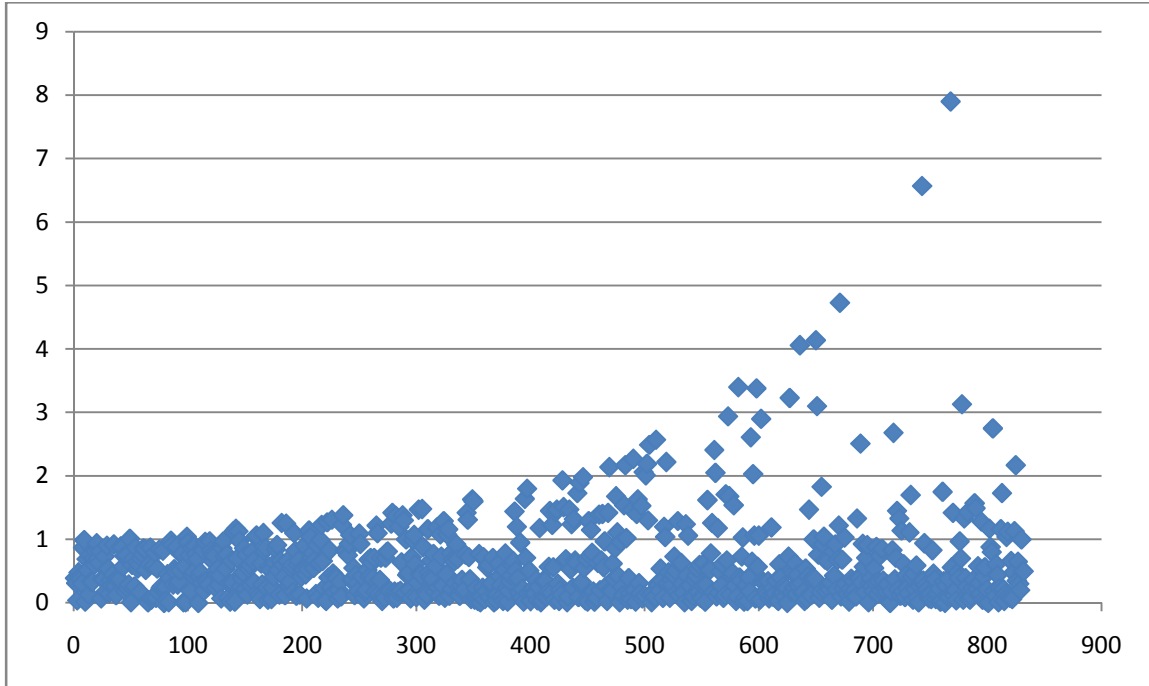
FIGURE 4: Increasing Dispersion of Price Deltas, 1,000 Customers

Figure 5 is a scatter plot from a simulation run where $n = 36,000$ and T turns out to be $30,832$. The substantially hyperbolic boundary is clearly delineated.
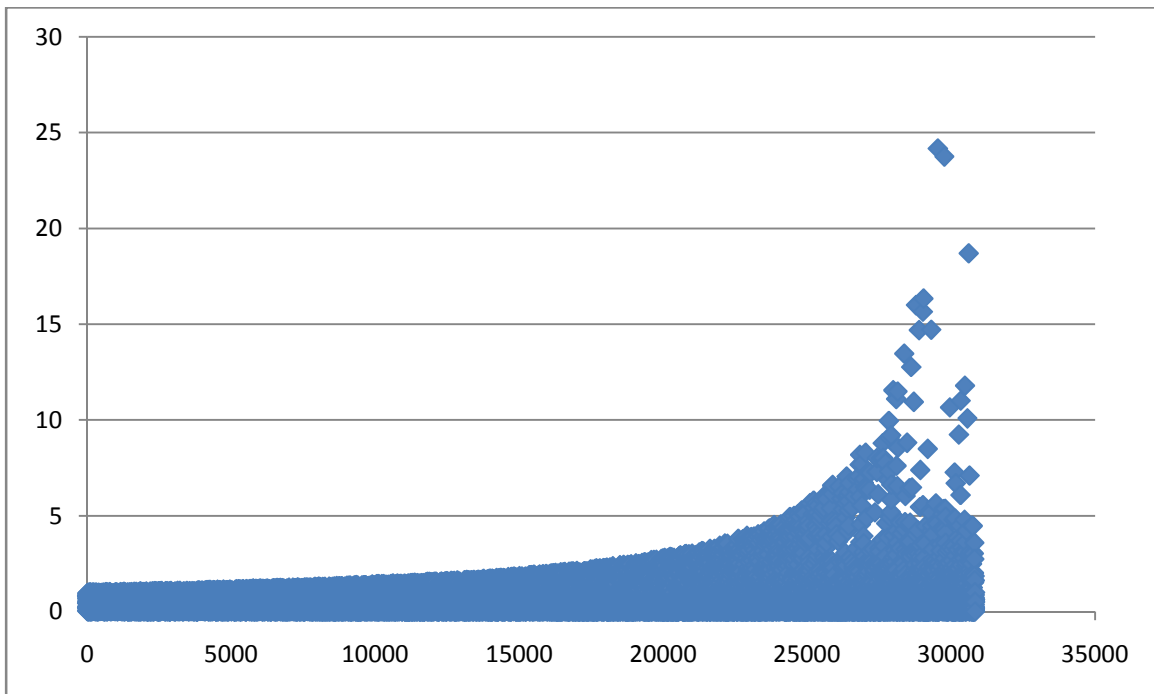

FIGURE 5: Increasing Dispersion of Price Deltas, 36,000 Customers

Figures 6 and 7 show actual data from the $n = 1,000$ simulation run: price deltas can be nearly infinitesimal, or can be notable. Most of the deltas are small, reflecting the fact that the defecting customer has utilization near the mean, and therefore doesn't impact it very much, and/or that there is a large population of customers, so a single defection in or out won't make much of an impact.
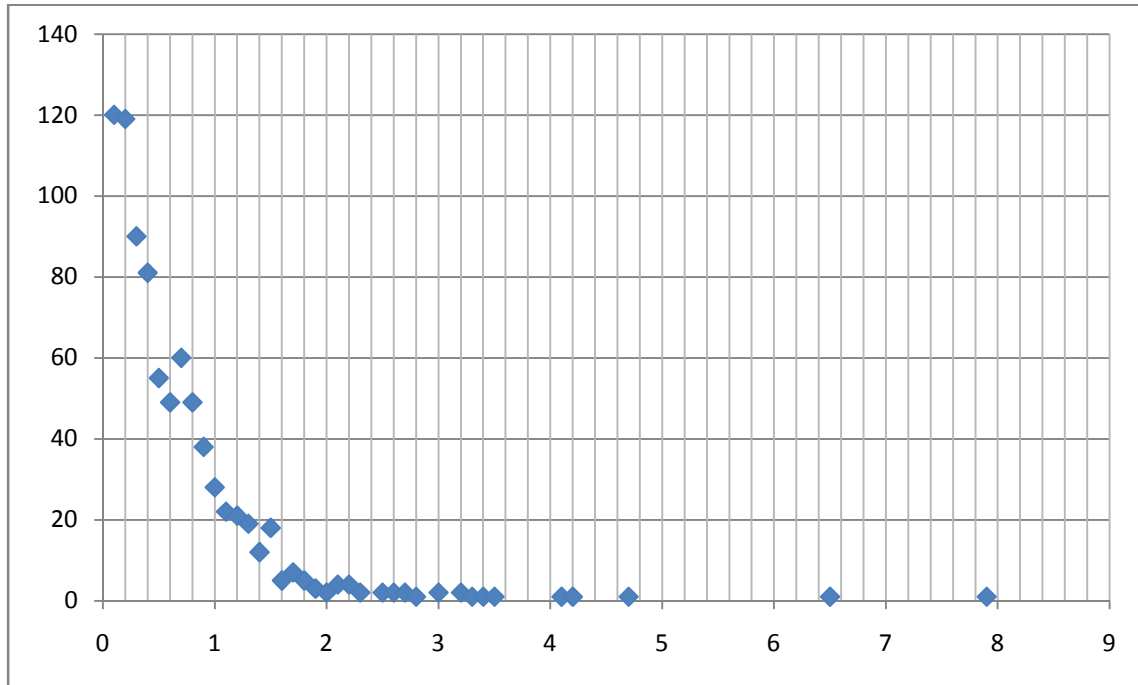
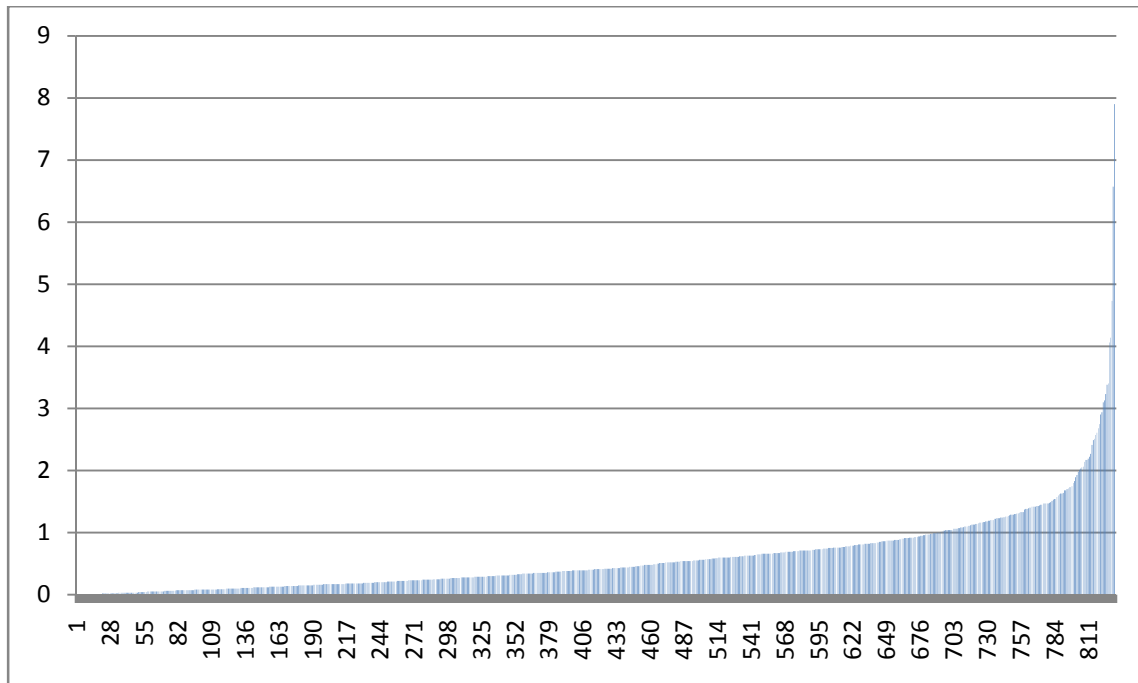**FIGURE 6: Histogram of Price Deltas in .1 Increment Buckets**

FIGURE 7: Sorted Price Deltas For n=1,000 Simulation Run

4.2. DEFECTION PROBABILITY

If the consumption pattern is uniformly distributed and with a large enough number of customers randomly distributed, we can expect the following to occur. First, given that $|C| = n$ is large, the law of large numbers suggests that the customers will initially be pretty close to evenly distributed between the two providers. At first, it is equally likely that a customer will defect from a flat-rate to a usage-based plan as it is that the reverse will happen, i.e., $p(A \rightarrow B) = p(B \rightarrow A)$. However, as we near the terminal state, it becomes increasingly unlikely for an $A \rightarrow B$ transition to happen. Thus, if we denote the probability of a transition from A to B at time t as $p_t(A \rightarrow B)$, we can see that at the extrema it at least roughly follows $p_t(A \rightarrow B) \approx \frac{T-t}{2T}$ and conversely, that $p_t(B \rightarrow A) \approx 1 - \frac{T-t}{2T}$. Simulation results show that $A \rightarrow B$ transitions are exceedingly rare for a substantial period leading up to T. In other words, once a critical point is reached, customers leave the flat-rate plan in droves, with very few swimming against this current.

In the example simulation run, we can examine the net gain of usage-based customers. A $B \rightarrow A$ transition is then a "$+1$", and an $A \rightarrow B$ transition would be a "-1". We can see the even balance in the early steps gradually give way to a dominant preference for the last third of the simulation for defections to pay-per-use:
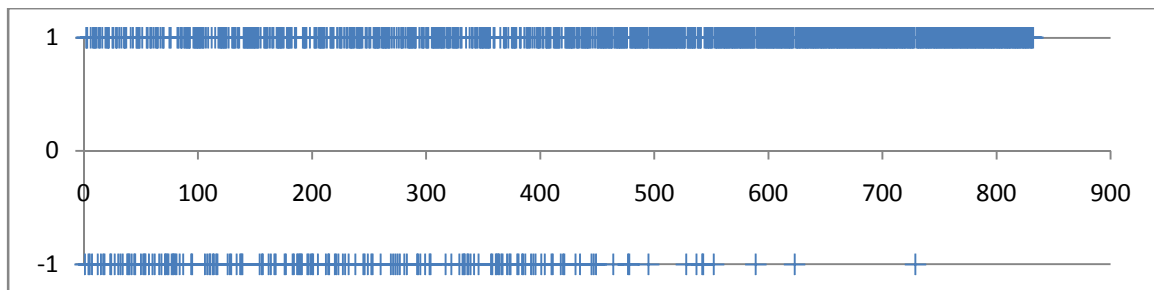
FIGURE 8: Net Gain (Loss) of Usage-Based Customers Each Step

As there must be a transition in every step until the terminal state, we can expect that the upper right should be a solid bar punctuated only by extremely small gaps from the increasingly rare $A \rightarrow B$ defections, which in this simulation run occurred in steps 589, 623, and 729. It may be somewhat hard to see the scatter plot entries, so another way to view this phase change is via the net number of à la carte users, $|A|$ over time, in Figure 9:

FIGURE 9: Number of Usage-Based Customers Each Step, n=1,000

Figure 10 shows a smoother curve based on a simulation run where $n = 36,000$.

FIGURE 10: Number of Usage-Based Customers Each Step, n=36,000

Examining the chart it will be appreciated that the curve is relatively flat in early steps, in other words, the size $|\boldsymbol{A}|$ does not change much, reflecting a roughly equal disposition of $\boldsymbol{B} \rightarrow \boldsymbol{A}$ transitions as $\boldsymbol{A} \rightarrow \boldsymbol{B}$ transitions. The tide gradually turns, eventually represents an apparently steady flow of defections to $\boldsymbol{A}$.
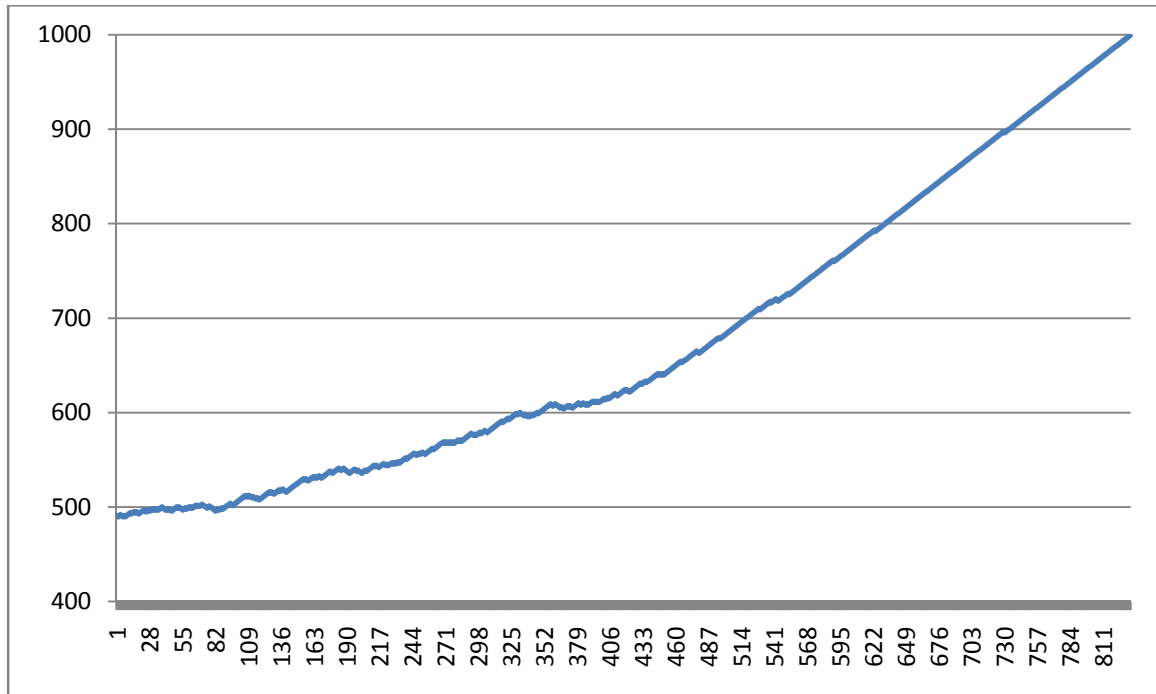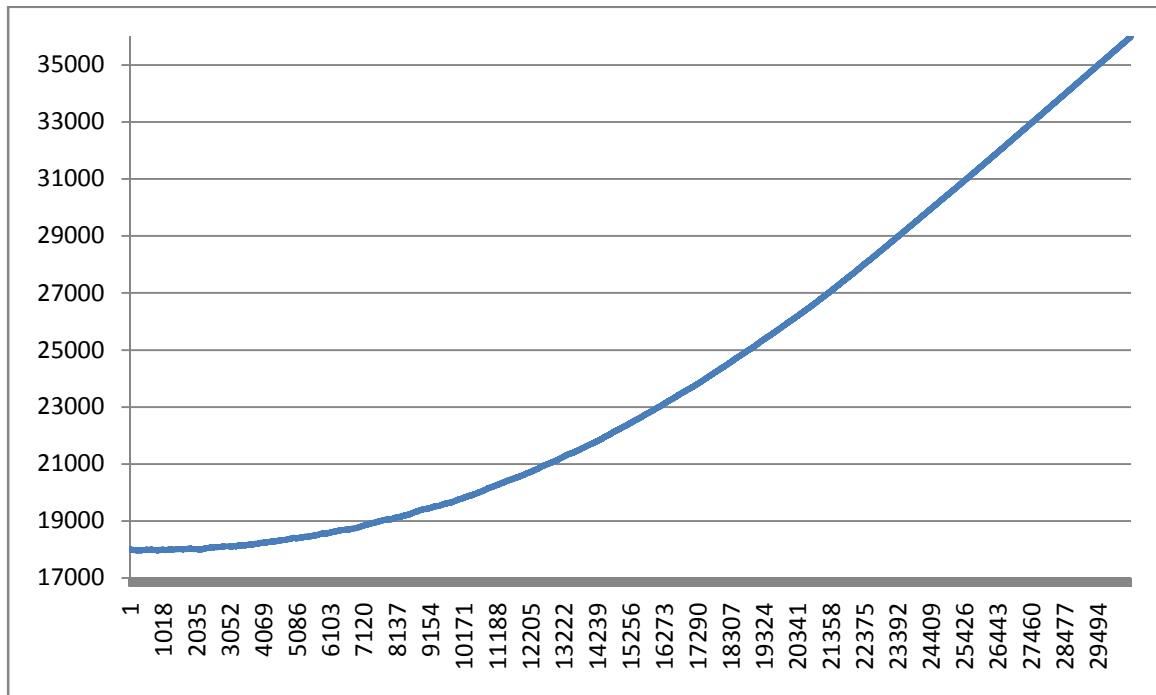
Simultaneous defections accelerate this process. At the extreme, suppose all customers that *can* defect in a given step *do*. Let's assume that the customers' consumption levels are uniformly distributed on the interval $[0, n]$. At $t = 0$, we can expect that half of the customers belong to each provider, i.e., $E(|\boldsymbol{A}|) = E(|\boldsymbol{B}|) = \frac{|C|}{2} = \frac{n}{2}$. Also, the expected value of the price $E(\boldsymbol{P}_{i,B,0})$ is at the midpoint of the interval, $n/2$. Consequently, at the next step, half the customers, which are the heavier than average consumers, will all simultaneously defect to the flat-rate plan, if they aren't already there. The "bottom" half—the lighter than average customers—will all simultaneously defect to the *à la carte* plan. As a result, $E(\boldsymbol{P}_{i,B,1}) = n \times 3/4$, which is the midpoint of the interval $\left[\frac{n}{2}, n\right]$. Customers with lower usage than this will simultaneously defect, and this *tâtonnement* will continue, so that $E\left(\boldsymbol{P}_{i,B,t}\right) = n \times \left(1 - \left(\frac{1}{2}\right)^{t+1}\right)$. This results in an accelerated convergence of $\boldsymbol{P}_{i,B,t}$ to $c_{max} \times M$ in only $O(\log_2 n)$ steps. As may be expected, for a uniform distribution of consumption levels, if the number of simultaneous defections is somewhere between 1 and "all possible," the terminal state T will be achieved in somewhere between $\log_2 n$ and $2n$ steps.

We need to distinguish between expected duration of the process for uniformly distributed customer usage, and worst case, as it is also possible to construct pathological usage distributions, where only one customer can defect even when simultaneous defections are allowed. For example, consider a customer set with usage levels constructed as follows. Select a positive interval α and a positive (if infinitesimal) increment ε. Let the heaviest consumer c_{max} have consumption $\boldsymbol{U}(c_{max})$. Let the next customer have consumption $\boldsymbol{U}(c_{max}) - \alpha$. Now let the next customer X_1 have usage $\boldsymbol{U}(X_1)$ light enough that the average consumption including it is less than $\boldsymbol{U}(c_{max}) - \alpha$. For this to be the case, it must be the case that

$$\frac{\boldsymbol{U}(c_{max}) + (\boldsymbol{U}(c_{max}) - \alpha) + \boldsymbol{U}(X_1)}{3} < (\boldsymbol{U}(c_{max}) - \alpha)$$

Or, rewriting terms:

$$\boldsymbol{U}(c_{max}) + \boldsymbol{U}(c_{max}) - \alpha + \boldsymbol{U}(X_1) < 3\boldsymbol{U}(c_{max}) - 3\alpha$$

Simplifying, we have

$$\boldsymbol{U}(X_1) < \boldsymbol{U}(c_{max}) - 2\alpha$$

We can achieve this by setting $U(X_1) + \varepsilon = U(c_{max}) - 2\alpha$. Now let there be an X_2 with $U(X_2)$ less than the average of $U(c_{max})$, $U(c_{max}) - \alpha$, and $U(X_1)$. Then

$$\frac{U(c_{max}) + (U(c_{max}) - \alpha) + U(X_1) + U(X2)}{4} < U(X_1)$$

Then, substituting for $U(X_1)$ using the equality $U(X_1) = U(c_{max}) - \varepsilon - 2\alpha$:

$$U(c_{max}) - 3\alpha + U(X_2) < U(c_{max}) - 4\varepsilon - 8\alpha$$

Therefore:

$$U(X_2) < U(c_{max}) - 4\varepsilon - 5\alpha$$

In fact, let us assume that $U(X_2)$ differs by ε as well, and thus let

$$U(X_2) + \varepsilon = U(c_{max}) - 4\varepsilon - 5\alpha$$

Simplifying,

$$U(X_2) = U(c_{max}) - 5\varepsilon - 5\alpha$$

We can continue this process as long as we would like, always making sure that the new lowest-consuming customer has consumption less than the average of all customers including itself. Of course, we must make sure that $U(c_{max})$ is large enough—or, equivalently, that α and ε are small enough—that the least consuming customer has at least a modicum of positive consumption. As an example, let $U(c_{max}) = 100$, $\alpha = 10$, and $\varepsilon = 1$. Then the four customer usage levels created via this process are 45, 79, 90, and 100. Note that if these four customers are $\in B_0$, then $\mu(U(B_0) = (45 + 79 + 90 + 100)/4$, which is 78.5. Consequently, only the lightest customer can defect. Then, $\mu(U(B_1)$ is $(79 + 90 + 100) / 3$, which is 89.666, Consequently, again only the lightest user can defect.

5. NON-RANDOM SELECTION, MULTIPLE PRODUCTS, DEFECTION FREQUENCY, RE-PRICING FREQUENCY, DEMAND VARIATION, CAPACITY CONSTRAINTS AND OTHER SCENARIOS

In the base case, there are a number of customers and only two providers, one of each kind. Clearly, there are many variations possible. For example, there may only be one provider: a monopoly. This is often the case in countries with a

single post, telephone, and telegraph (PTT) company. Or, there may only be one customer: a monopsony as in the U.S. market for space stations. Or, there may be a long-term relationship, as between a live in nanny and the host family, with several, but not numerous, re-pricing opportunities. Or the market may have millions of buyers with thousands of re-pricing opportunities every day, e.g., the equities market for blue chip stocks.

Also, in the base case we examined, demand varies between customers, but not intertemporally. However, demand can often vary as a completely stochastic process following a particular distribution, e.g., demand might be normally or uniformly distributed across an interval, or it may be a stateful (even though possibly memoryless) discrete random process such as a Markov chain, where the current state is important in determining the transition probabilities to a set of next states. For example, from tenth grade in High School, there is a high likelihood of a transition to eleventh grade, a hopefully lower probability of dropping out, and perhaps non-zero probabilities of skipping to twelfth grade or college. However, there is an extremely low probability of transitioning from kindergarten to graduate school. Characterizing demand variability is important, especially under information asymmetry. The reason is as follows.

Suppose that a customer's demand in the current time period is low. She may then be tempted to defect from a flat-rate to consumption-based plan. However, if in the next time period her demand is extremely high, that strategy will not be effective. It is like switching lines at the grocery store cash/wrap repeatedly, only to find out that you would have been better off where you were. However, if there is limited variation between time periods, an extremely low demand this period is an indicator of below average demand in the next few periods, so defecting to a usage-based plan is rational.

Perhaps counter-intuitively, **there are conditions under which demand for each user varies across a wide range of lowest usage and highest usage, yet flat-rate pricing is unsustainable** under conditions as we've described. The reason is that low-usage users still defect away from flat-rate plans, and high-usage users still defect to them. If there is any dependence on current state, the average price for the flat-rate plan will still move up. For a prosaic example, suppose you are a glutton during half of the weeks of the year and on a diet during the other twenty-six weeks. It would make sense to visit the all-you-can-eat buffet when pigging out, and order *à la carte* the rest of the time. If everyone adopts this strategy, the buffets can't price for the average, only for the average glutton.

5.1. NON-RANDOM INITIAL STATES AND DEFECTION ORDER PROTOCOLS

The sequential defection rule specifies that a customer d in the set D_t be selected at random to defect. It is easy to envision all sorts of specific protocols that further specify how the rule is to be applied, however, such as "the consumer with the most egregious overcharging must defect first", that "flat-rate to *à la carte* defections must occur before"—or after—*à la carte* to flat-rate defections, that "defections must alternate if possible", that at time 0 a specific allocation of customers must be made, e.g., "all customers in flat-rate," "as close to 50% as possible", "lightest customers must start in flat-rate" or must start in *à la carte*, etc. None of this matters in the slightest, as the following corollaries highlight:

Proposition 11: Theorem 1 holds regardless of initial state (given at least one customer on a flat-rate plan).

> **Proof**: Since Theorem 1 holds for *any* (random) initial state, it also holds for specific initial states conforming to some rule. ∎

Proposition 12: Theorem 1 holds regardless of defection order.

> **Proof**: As above, since Theorem 1 holds for <u>all</u> random sequential defection orders, it also holds for a subset of those defection orders conforming to some protocol. ∎

5.2. INITIAL OFFER PRICES FOR FLAT-RATE PLANS

The reason that we need at least one customer on a flat-rate plan is to define an initial price $P_{i,B,0}$ at time 0. Conceptually, however, we certainly can specify prices some other way for the initial state. For example, the price might be free, whenever there are no customers ($|B_t| = 0 \rightarrow P_{i,B,t} = 0$), or set to a specific initial value.

Proposition 13: Theorem 1 holds even if there are no customers for the flat-rate plan in the initial state and the initial price is free, that is, $|B_0| = 0$ and $P_{i,B,0} = 0$.

> **Proof**: Briefly, $A_0 = C = D$, $B_0 = \emptyset$, and $P_{i,B,0} = 0$. In other words, all customers are eligible to defect, since they are paying more than 0 under the pay-per-use plan. Once any customer defects, we are in a state corresponding to the antecedent of Theorem 1, at which point the system dynamics will continue as before. ∎

Proposition 14: Theorem 1 holds if there are no customers for the flat-rate plan in the initial state and the initial price is less than what a heaviest user would pay, i.e., $|B_0| = 0$ and $P_{i,B,0} < U(c_i) \times M$.

> **Proof**: As above, since the heaviest users are patrons of A, they are members of D, since they are paying $U(c_i) \times M$ whereas if they defect they would pay $< U(c_i) \times M$. Since D is non-empty, there will be at least one defection, at which point we are in the state corresponding to an initial state for Theorem 1. ∎

Proposition 15: If there are no customers for the flat-rate plan in the initial state and the initial price is greater than or equal what a heaviest user would pay, i.e., $|B_0| = 0$ and $P_{i,B,0} \geq U(c_n) \times M$, then the initial state is the terminal state and all customers will remain on consumption-based plans.

> **Proof**: According to the sequential defection rule, a customer c_i can defect from A to B at time t, that is, $c_i \in D_t$, if and only if $P_{i,B,t} < P_{i,A,t}$. However, since $P_{i,B,0} \geq U(c_n) \times M$, there is no such customer, consequently no defections will occur. If no defections occur, $T = 0$ is the terminal state, and $A_0 = C = A_T$. ∎

5.3. MULTIPLE DEFECTIONS BEFORE REPRICING

Let us consider a modified defection rule that allows for multiple simultaneous defections before each re-pricing action, with state transitions as follows:

> **The Simultaneous Defection Rule:** Let D_t be the set of defection candidates at time t, that is, let $D_t = \{ c_i \in A_t, P_{i,B,t} < P_{i,A,t} \} \cup \{ c_i \in B_t, P_{i,A,t} < P_{i,B,t} \}$. Select one <u>or more</u> customers $d_1, d_2, d_3, \dots \in D_t$ at random and switch their provider, so that for any selected d_i, either $d_i \in A_t$ and $d_i \in B_{t+1}$, or $d_i \in B_t$ and $d_i \in A_{t+1}$.

Again D_t is the subset of customers who could get a better deal at time t, and since they are active, rational, maximizers, the <u>simultaneous</u> defection rule selects <u>one or more</u> of those customers (at random) and switches them. The difference from the original sequential defection rule is that there may be multiple defections before each re-pricing action. We can interpret this as that the frequency of re-pricing remains the same but multiple customers simultaneously defect, or perhaps as that customers still defect sequentially, but re-pricing is only done "sporadically." We just highlight some of the key propositions that still apply under this rule.

Proposition 3b: Let there be at least two customers on the all-you-can-eat plan: $c_i, c_j \in B_t,\ i \neq j$, with different consumption levels, i.e., $U(c_i) \neq U(c_j)$. Then, using the simultaneous defection rule, there is at least one customer $c_i,\ 0 < i \leq n - h,$ that can defect to the à la carte plan.

> **Proof** : According to Proposition 3, if there are at least two customers with different consumption levels under the sequential defection rule, one can defect. But if *one* can defect under the *sequential* rule, then *at least one* can defect under the *simultaneous* rule. ∎

Proposition 6b: Using the simultaneous defection rule, $P_{i,B,t} < P_{i,B,t+1},\ 0 \leq t < T$, that is, the all-you-can-eat price monotonically increases, regardless of whether a customer defects from all-you-can-eat to *à la carte* or from *à la carte* to all-you-can-eat.

> **Proof (outline)**: In line with the more formal proof provided for Proposition 6, since only heavier-than-average users defect from A to B, and only lighter than average users defect from B to A, the average consumption of B can only increase and therefore the price can only increase with each pricing action. ∎

Theorem 1b: In a market where the Simultaneous Defection Rule applies to define state transitions, the terminal state is no different than under the Sequential Defection Rule.

> **Proof:** Assume a given initial state, i.e., allocation of customers to providers, and sequence of simultaneous defections leading to a particular terminal state. For each simultaneous defection of k customers, we serialize it into k defections of 1 customer, comprising $A \to B$ defections arranged in order of increasing consumption followed by $B \to A$ defections (in any order). Consider each of the $A \to B$ defections in turn. Let the lightest such consumer be d_1. Since d_1 is eligible to defect under the Simultaneous Defection Rule, it certainly is under the Sequential Defection Rule. Now consider d_2. Since $\mu(U(B_t)) < U(d_1) \leq U(d_2)$, if d_2 was eligible to defect at the same time that d_1 was, then d_2 will be eligible to defect *after* d_1, since $\mu(U(B_{t+1})) < U(d_2) \mid d1 \in B_{t+1}$. We can continue this process, serializing each of the $A \to B$ defections. Now, since the $B \to A$ defections were feasible before these serialized defections occurred, they are certainly still feasible after the $A \to B$ defections have been conducted. Moreover, regardless of the order of $B \to A$ defections, the price will monotonically increase, consequently each defection candidate will be eligible to defect. In other words, while the $A \to B$ defections needed to occur first in the serialization process, and their order was important, we need not be as concerned with the $B \to A$ defections. After all simultaneous defections have been serialized, we have a sequence of defections that conforms to the Serial Defection Rule.

However, this sequence is a member of the set of all random sequences of defections under the Serial Defection Rule, so the terminal state cannot be different. ∎

Remember that we don't distinguish between whether heaviest users finish up at the flat-rate provider or the usage-based provider, so by "no different," we mean that all the non-heaviest users have migrated to the usage-based plan. Also, clearly the number of steps required and thus T will differ.

It may appear as if the Simultaneous Defection Rule changes nothing about our first set of propositions, 1 through 10, but there are some differences. For example, Proposition 8 no longer holds. To see this, suppose that B_0 comprises one customer c_1, with a consumption level of 10, while A_0 has two customers, say, c_2, with a consumption level of 11, and c_3 with a consumption level of 78. They both rationally defect at $t = 0$ so that B_1 now has an average consumption $\mu(U(B_1))$ of $33 = 99 (10 + 11 + 78) / 3$. Customer c_2, will now want to defect back immediately to A.

5.4.   MULTIPLE PROVIDERS

What happens when there are multiple "all-you-can-eat" buffet providers B_1, B_2, B_3, B_4, ... and/or multiple utility providers A_1, A_2, A_3, A_4, ...? Briefly, at any given time including the initial state, the price of any all-you-can-eat provider is a result of its customer set, and the specific consumption of each customer in that set. Consequently, the price for provider B_1 at time t, $P_{i,B_1,t}$, may be less than, equal to, or greater than that of provider B_2, $P_{i,B_2,t}$, and after a customer defects from B_1 to B_2, B_2 to B_1, or B_1 or B_2 to another flat-rate provider, say B_3 or usage-based provider, say A_1 or A_2, or *vice versa* this price relationship may change or stay the same. However, usage and therefore price still will increase monotonically for each provider, and lighter-than-average users will still want to defect somewhere else. Consequently, "light" users will sooner or later defect to an *à la carte* provider, which may cause more inter-flat-rate shuffles, but eventually only the set of identically heaviest users will remain with flat rate providers, and, as before, they will be paying the same price as if they were with *à la carte* providers. Moreover, in the shuffle, one or several flat-rate providers may lose all customers, at which point we will assume that they exit the market.

However, the Finite Termination lemma (Proposition 9) no longer applies:

Proposition 16 (Potential Infinite Churn): If the number of transitions is bounded, there is not more than one flat-rate provider.

Proof: We have already shown that in the case of one flat-rate provider, the number of transitions is bounded by $2 \times |C|$. We show that it is the only such case by counterexample, demonstrating that there is an

illustrative case with two flat-rate providers where the number of transitions is not bounded, and that extending to more flat-rate providers and more usage-based providers does not alter this lack of a bound. Without loss of generality, let M be 1. Consider the case of three providers: B_1, B_2, and A_1. Consider the following initial mapping of customers to providers: Provider B_1 has customers with usage levels of 1, 6, and 8. Provider B_2 has customers with usage levels of 1 and 8. Provider A_1 has no customers. The customer with usage 6 we will call c_w (for waffler). Customer c_w has $P_{w,B_1,0}$ of $(1+6+8)/3 = 5$. $P_{w,A_1,0} = 6$. But $P_{w,B_2,0} = (1+8)/2 = 4.5$. Consequently, c_w can defect to B_2 with non-zero probability (other defections are also possible). Let's assume he does so. Now, however, the tables are turned, where $P_{w,B_1,1} = 4.5$, and $P_{w,B_2,1} = 5$, and the usage-based price of $P_{w,A_1,1}$ is of course still 6. Consequently, c_w can defect back to B_1 with non-zero probability. It is easy to see that there is a non-zero probability of an infinite loop, where for all non-negative integers s, $P_{w,B_2,2s} < P_{w,B_1,2s} < P_{w,A_1,2s}$ and where $P_{w,B_1,2s+1} < P_{w,B_2,2s+1} < P_{w,A_1,2s+1}$. Thus, $c_w \in B_{1+(t \bmod 2)}$. Consequently, for any integer k there is a non-zero probability that $T > k$. Moreover, if the lack of a bound is true for b flat rate providers, it is also true for $b + 1$ flat-rate providers, since we can always add a new provider B_{b+1} either with one or more customers such that $\mu\big(U(B_{b+1})\big) > 5$, eliminating B_{b+1} from being a potential defection target and thus altering the probability, but not eliminating the possibility of a set of state transitions of any given length, or also with two customers with usage levels 1 and 8, making B_{b+1} another potential target for this round-robin process. Similarly, adding additional usage-based providers does not affect the outcome, since $P_{w,A_1,t} = P_{w,A_2,t} = P_{w,A_3,t}$. ∎

Of course, these are specially constructed cases, which behave essentially as memoryless Markov processes and whose survival time may be characterized via an exponential distribution. For non-pathological cases, we can expect a sequence of defections that terminates "not that much more" slowly than the single provider case. In this model, by the way, our standard observation that the heaviest user can not defect from a flat-rate plan no longer holds. Any user, including a heaviest one, can defect from a more costly to a less costly plan, and so, while a heaviest user will never defect from a flat-rate plan to a usage-based one, at any given time, different flat-rate plans may have different μs and therefore prices and therefore may be viable defection targets.

5.5.   ONE FLAT-RATE PROVIDER, ELASTIC DEMAND

Numerous examples and models exist where individual actors make rational choices that turn out to reduce their utility. The classic Prisoner's Dilemma is an example, and restatements or related problems include the Tragedy of the Commons[lxvii], where every herder always has an incentive to add another animal to a common pasture, because the benefit is gained solely by the herder and the cost is borne across all of them, the ("unscrupulous") "Diner's Dilemma," where every diner has an incentive to order a substantially more expensive dish that is only slightly tastier when the check will be shared by all, leading them all to do so in which case they all pay the higher price and therefore reduce their utility, the "free rider" problem where subway or bus riders jump the turnstile. Arms races, where additional expenditures do not lead to increased security, are another example. It may be argued that all these problems may be traced to issues in relating marginal cost to global effects.

Empirical results have also confirmed that customer behavior can be influenced by service characteristics: shifting to usage-based plans ("measured telephone service") can reduce calling and/or reduce calls made during more expensive calling times.[lxviii]

Not only can consumption rise, but there can be non-linear effects that emerge from such increased consumption. For example, *congestion externalities*[lxix] arise as follows: flat-rate pricing means that the marginal cost to a user of marginal consumption is zero. Therefore, utilization will rise until some natural limit is reached, e.g., there can be no more than 24 hours of Internet access by a user in any given day, nor more bandwidth consumed than the physical limit of the transmission line / data network access and transport limits. Systems, such as computer servers and network equipment can degrade in performance rapidly once a threshold is reached. These effects are not restricted to digital equipment, consider a highway, where as more and more vehicles enter the throughput increases linearly, until a critical threshold is reached and congestion causes traffic jams.

With linear usage-based plans, the marginal price for one more unit is M. However, with flat-rate plans, the marginal price is 0. Suppose there is one flat-rate provider but there are no pay-per-use plans available and thus defection is not an option? One challenge that Nahata *et al* observe[lxx] is that of pricing for profit maximization. They consider a simple case of low-demand (and therefore low willingness to pay) customers and high-demand (and therefore high willingness to pay) customers. If the all-you-can eat price $P_{i,B,t}$ is set low enough to be attractive to the low-demand customers, the high-demand customers will also be attracted, and the market size is maximized. However, there is an opportunity cost, as high-demand customers will be paying the lower price. On the other hand, if the price is set higher, then low-usage customers may not buy at all, but revenue and profitability from the high-demand segment will be maximized. Clearly, the optimal price depends on the size of each segment and profitability at both price-points.

With marginal price of zero, there appears to be little to restrain a user to be parsimonious in consumption, so demand rises to a level of maximal satiation, unbounded by economic concerns. For example, the INternet Demand EXperiment (INDEX[lxxi]) showed that individual users increased bandwidth or total data transferred consumption by 2, 3, or up to 10 times under flat-rate plans than charge-by-minute or charge-by-byte-transferred plans.[lxxii] Moreover, flat-rate usage was almost as high as free usage: once customers paid the flat-rate, they (surprisingly, given the "sunk-cost fallacy" cognitive bias) correctly treated it as a sunk cost and consumed virtually as much as without any "front-end" charge.

However, another rational response for the low-usage customers might be to consume enough to realize the value inherent in the flat rate. Of course, if all customers do this, average consumption increases, therefore the flat-rate price must increase, in turn causing further consumption. In practice, there are two distinct drivers resulting in the same outcome. One driver is when customers are incented to increase consumption to better align price paid with amount consumed, the other is when there is no disincentive to limit consumption.

Just and Wansink show that such effects do happen[lxxiii]. In studying fixed-price plans at a pizzeria, they empirically demonstrated that increasing the price paid increased consumption, "due to a desire to get a 'good deal'". In fact, they found that "individuals are significantly motivated by a desire to get their money's worth." Logically, reducing the price paid decreased consumption. They observe that this is an example of the "sunk cost fallacy," one of many behavioral economic cognitive biases. Specifically, once a fixed price has been paid for the pizza, the marginal utility to a pizza eater of consumption of slices should not depend on the price paid. However, empirically it does. They also show that additional consumption does not lead to greater happiness. There may be decreasing marginal utility or even disutility such as malaise (Alka-Seltzer's "I can't believe I ate the whole thing") or obesity.

Odlyzko[lxxiv] observes that any "kind of barrier to usage, such as explicit payment [under usage-based plans], serves to discourage usage," whereas that a "general rule of thumb is that switching from metered to flat-rate pricing increases usage by 50 to 200 percent." As an example of the former, it has been suggested that Pay-As-You-Drive (PAYD) insurance, in which premiums, rather than being flat-rate, are correlated to miles driven (and context, as well, e.g., traffic levels, road conditions) are likely to increase driving by infrequent drivers and reduce driving by frequent drivers[lxxv], and may be the most effective way of decreasing gasoline consumption—rather than say, gasoline taxes.[lxxvi]

The flip side of an incentive to consume is lack of a disincentive to overconsume. Cocchi *et al*, in exploring flat-rate vs. priority pricing, observe that "under the flat pricing scheme, it is clear that each application type will choose to request high priority service. This is because there is no monetary incentive to request low

priority service and, if there is any congestion in the network, there is a performance incentive to request high priority service."[lxxvii]   An example of this is when AOL first introduced flat-rate pricing. As there was no marginal cost for longer-duration connectivity, users had no disincentive to "log out." As a result, "usage"—measured by monthly connect time— more than tripled within a year.[lxxviii] One might say that such a result was not necessarily a shift in usage, in that a user who is connected but without a networked application running such as an email client or a browser is not really "using" anything, but for the technology of the time—dial-up modem pools—a connected but idle user used just as many access resources as a connected active user.

Other unfortunate effects can happen, e.g., MacKie-Mason and Varian[lxxix] describe what they term a "Yogi Berra equilibrium," after his famous remark "it's so crowded that no one goes there anymore," to describe a state in which the available resources are dominated by congestion-tolerant users. Because they are tolerant, they have a low willingness to pay for capacity investments to reduce congestion, perpetuating the equilibrium.

If customers are content to "receive their fair share" by increasing consumption to reflect payment, it is easy to see that everyone's consumption will rise asymptotically to equal that of the heaviest user, subject to other constraints (the amount of food or alcohol one can consume in any given time period at an all-inclusive resort, the amount of data that can be downloaded over a fixed wireline or mobile connection, etc.)

If there is competition, as in the consumption of executive pay by CEOs (which we might call "The Market for (Andrew) Mellons"), movie stars ("The Market for (Jack) Lemmons"), or sports stars ("The Market for (Roger) Clemens"), where status and ego are at stake, or in the case of consumption of yachts or homes by Hollywood celebrities, it is also easy to see that everyone's consumption will rise without intrinsic bound until an extrinsic factor (Congressional action, labor agreements, substitutions from outside the system, etc.) cause limits, at least in the short term.

The zero-price effect[lxxx] also becomes important. After all, if the marginal price for increased consumption is zero, this causes wasteful behavior, moral hazard, the tragedy of the commons. As one Internet user described when commenting on the transition from pay-per-minute dial-up plan to flat-rate broadband access: "What I like about it, and it is purely psychological, is that I pay a flat rate and I don't have to worry how long I am on the Internet. … So I just don't care, when I want to go on and surf, I surf. And it could be an hour, two hours, three hours, it doesn't matter."[lxxxi]

We formalize this discussion by allowing $U(\ )$, the consumption function, to vary over time as $U_t(\ )$.

Proposition 16: In an ecosystem with a single flat-rate provider B, if customers with less than "fair" payment for their consumption increase their consumption to the current average, all customers will, in the limit, have identical "heaviest" consumption, equivalent to the heaviest consumer at time 0. Formally, if $\forall i, 1 \leq i \leq n, U_t(c_i) < \mu(U(B_t)) \Rightarrow U_{t+1}(c_i) = \mu(U(B_t))$, then

$$\forall i, 1 \leq i \leq n, \lim_{t \to \infty} U_t(c_i) = U_0(c_{max})$$

Proof : We need to show that for any customer with less than maximal consumption, consumption will increase, but never exceed $U_0(c_{max})$. Suppose at time $t = 0$, all customers have identical consumption equivalent to c_{max}. Then $\mu(U(B_0)) = c_{max}$, so there is no customer c_i such that $U_t(c_i) < \mu(U(B_t))$. Therefore, we have reached a terminal state where consumption does not change, and where $\forall i, 1 \leq i \leq n$, $U_0(c_i) = U_0(c_{max})$, so it is true in the limit, since for any function $f(x) = k$, where k is a constant, $\lim_{x \to \infty} f(x) = k$.

However, suppose that not all customers have identical consumption at time $t = 0$. If not all customers have identical consumption at time t then there is a customer with minimal usage, which we'll denote by c_{min}.

Let us consider a special case first, where there are p such customers, $c_1, c_2, \ldots c_p$ with minimal usage, and q customers $c_{p+1}, c_{p+2}, \ldots c_{p+q}$ with maximal usage. We observe that there is no mechanism for maximal usage customers to increase their usage, consequently, for $\forall i, p + 1 \leq i \leq p + q, 0 < t, U_0(c_i) = U_t(c_i) = U_t(c_{max})$. Also, note that the total number of customers $|C| = |B_t| = p + q = n$. At time $t = 0$,

$$\mu(U(B_0)) = \frac{p \times U_0(c_{min}) + q \times U_0(c_{max})}{p + q}.$$

Clearly, the p customers with usage $U_0(c_{min})$ satisfy the condition $U_0(c_{min}) < \mu(U(B_0))$, therefore, applying the recursion, we set $U_1(c_{min}) = \mu(U(B_0))$,. In fact, we keep applying the recursion, so that:

$$U_{t+1}(c_{min}) = \mu(U(B_t)) = \frac{p \times U_t(c_{min}) + q \times U_0(c_{max})}{p + q}$$

This recursion generates a sequence that by its construction is monotonically increasing and bounded, therefore, by the monotone convergence theorem is convergent. We now need to determine its limit, L. Substituting L for $U_{t+1}(c_{min})$ and $U_t(c_{min})$, we have that:

$$L = \frac{p \times L + q \times U_0(c_{max})}{p + q}$$

Multiplying both sides by $(p + q)$, we get that:

$$p \times L + q \times L = p \times L + q \times U_0(c_{max})$$

Eliminating the $p \times L$ and dividing by q provides our desired result:

$$L = \lim_{t \to \infty} U_t(c_{min}) = U_0(c_{max})$$

When there are additional customers "between" c_{min} and c_{max}, we assert (without detailed proof) that this result should not change, as for any c_{mid} where $c_{min} < c_{mid} < c_{max}$, either $U_t(c_{min}) < U_t(c_{mid}) < \mu(U(B_t))$, in which case at $t + 1$ we will have $U_{t+1}(c_{min}) = U_{t+1}(c_{mid})$, which then equates to the prior case, or $U_t(c_{min}) < \mu(U(B_t)) \le U_t(c_{mid})$, in which case $U_t(c_{mid})$ will not be impacted on that step. Sooner or later, however, there will be a step t where c_{mid} is swept up by the rise of c_{min}. ∎

Proposition 17: In an ecosystem with a single flat-rate provider **B**, if customers with less than or equal to "fair" payment for their consumption increase their consumption to *greater than* the current average, all customers will increase their consumption without bound and there is no terminal state. Formally, if $\forall i, 1 \le i \le n, U_t(c_i) \le \mu(U(B_t)) \Rightarrow U_{t+1}(c_i) = \mu(U(B_t)) + \delta, \delta > 0$, then:

$$\forall i, 1 \le i \le n, \lim_{t \to \infty} U_t(c_i) = \infty$$

Proof : We simply observe that that if $\mu(U(B_t))$ is the average consumption level of **B** at time t, and $|B_t| > 0$, there must be at least one customer at or below this average. Consequently, there is at least one customer who will increase their consumption by at least δ. Consequently, in each time period, at least one customer increases by at least δ, and no customers reduce their consumption. Consequently, the average must increase by at least $\frac{\delta}{|B_t|}$, hence $\mu(U(B_{t+1})) \ge \mu(U(B_t)) + \frac{\delta}{|B_t|}$. Writing out these relationships, we have:

$$\underbrace{\mu(U(B_t)) \ge \mu(U(B_{t-1})) + \frac{\delta}{|B_t|} \ge \mu(U(B_{t-2})) + \frac{\delta}{|B_t|} \ge \cdots \ge \mu(U(B_0)) + \frac{\delta}{|B_t|}}_{t \text{ times}}$$

Thus,

$$\mu\big(U(B_t)\big) \geq \mu\big(U(B_0)\big) + \frac{\delta}{|B_t|} \times t$$

But then, of course, since $\delta > 0$ and $|B_t| > 0$ but finite and bounded by the constant $|C|$, we know $\lim\limits_{t \to \infty} \frac{\delta}{|B_t|} \times t = \infty$. Therefore, the mean consumption grows without bound, i.e.:

$$\lim\limits_{t \to \infty} \mu\big(U(B_t)\big) = \infty$$

And, no customer can be "left behind," because $\forall i, 1 \leq i \leq n, U_t(c_i) \leq \mu\big(U(B_t)\big) \Rightarrow U_{t+1}(c_i) = \mu\big(U(B_t)\big) + \delta$, so therefore

$$\forall i, 1 \leq i \leq n, \lim\limits_{t \to \infty} U_t(c_i) \geq \lim_{t \to \infty} \mu\big(U(B_t)\big) = \infty \ \blacksquare$$

Such effects are more than theoretical. Bubbles such as the Dutch Tulip mania are examples of uncontrolled escalation, as well as the Bazerman Auction, where a $20 bill is auctioned off but while the winner pays the amount bid the second-highest bidder still must pay while receiving nothing[lxxxii]. In "Why Has CEO Pay Increased So Much?," [lxxxiii] the authors propose several factors. One is "contagion," that is, the tendency for higher pay to be replicated. As the authors show, "if 10% of firms want to pay their CEO only half as much as their competitors, then the compensation of all CEOs decreases by 9%. However, if 10% of firms want to pay their CEO twice as much as their competitors, then the compensation of all CEO's doubles." Moreover, the coefficients and assumptions are key, otherwise, as the authors state, "there is no equilibrium with finite salaries," in other words, the pay race doesn't end until everyone is paid an infinite amount.

5.6. OTHER SCENARIOS

There are numerous other scenarios for which space does not permit full investigation, consequently we merely outline some arguments.

Suppose that in the short term (or any relevant time frame) there is fixed production capacity, and there is an iterative demand function as described above? Then we end up with everyone attempting to consume more, but no one receiving more. Consequently, no one is better off. These things happen all the time: consider broadband internet wireless data plans and network congestion. Such increases in consumption also may be due to exogenous factors, such as technology evolution and usability enhancements. A good example is smartphone adoption, which has caused dramatic increases in network service

provider bandwidth requirements. And, as one service provider CEO has been quoted: "If this Title 2 regulation looks imminent, we have to re-evaluate whether we put shovels in the ground."[lxxxiv] In other words, without a clear path to monetizing capital investments in network infrastructure, those investments would have to be re-evaluated.

In such a case of finite, fixed capacity Y, if n customers increase their consumption to c_{max}, as in Proposition 16, or are spiraling upward limitlessly as in Proposition 17, it is clear that, after time t where the average consumption demanded $\mu(\boldsymbol{U}(\boldsymbol{B}_t)) \geq Y/n$ only an expected value of Y/n worth of consumable capacity will actually be delivered to any customer.

Pricing, as Courcoubetis and Weber have succinctly stated, is "a mechanism to regulate access to network resources and restrict congestion to an acceptable level."[lxxxv] To ensure the ability to meet service level objectives, there either must be more than enough capacity in aggregate, or during peak usage periods there must be a means to determine who gets it. In data networks, "differential services" and "integrated services" are examples of strategies for doing this. The differential services approach divides customers or the applications that those customers have into different priorities. For example, an application of a highway might be to transport kids to a movie, or to transport a trauma victim to a hospital. In other contexts, such prioritization and differential service might enable price discrimination, e.g., charging more for first class than for coach. Protocols in use on highways give priority to the ambulance. Integrated services are more like traffic lights that control access to highways, only allowing new vehicles in if the flow of traffic is above a minimum threshold.

Note also that there may be interesting effects with services whose capital-intensive resources are subject to alternating periods of sufficient capacity and congestion. MacKie-Mason and Varian[lxxxvi] argue that when underutilized, the marginal cost of a unit of service is near zero, but when congested, market mechanisms such as congestion pricing may be helpful. They suggest a pricing plan where the price is free when the resources are less than 100% utilized, but then a congestion fee to help internalize the externalities inherent in congestion, where one user's action can impose a cost on another user.

Congestion fees can create an interesting equilibrium: if users' consumption rises when services are flat rate, this causes congestion. Then, if the marginal utility of additional consumption is in fact minimal, customers will avoid paying the congestion fee by reducing usage. But then the system will always run "hot," i.e., near full capacity.

Let's consider another variation, where instead of consuming a quantity of a single product, customers actually consume multiple ones. These may not necessarily be sold as a pure bundle, they just represent a multidimensional vector of consumption. Formally, there is a set of q products $\boldsymbol{P} = \{p_1, p_2, \ldots p_q\}$,

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and for each customer   product   there is a consumption function   . As before, assume that consumption is fixed intertemporally for a given customer, but that there is a dispersed distribution across customers. Let us, for the sake of simplicity assume that consumption, instead of in units, is in a common unit such as dollars. As before, we can expect that the flat-rate is set based on the average consumption of the customers of the flat-rate provider, but then there will be customers who are relatively underpaying and others who are overpaying. We need to convert what was a single dimension into two or more dimensions. Graphically, we can view the difference as in Figure 11.
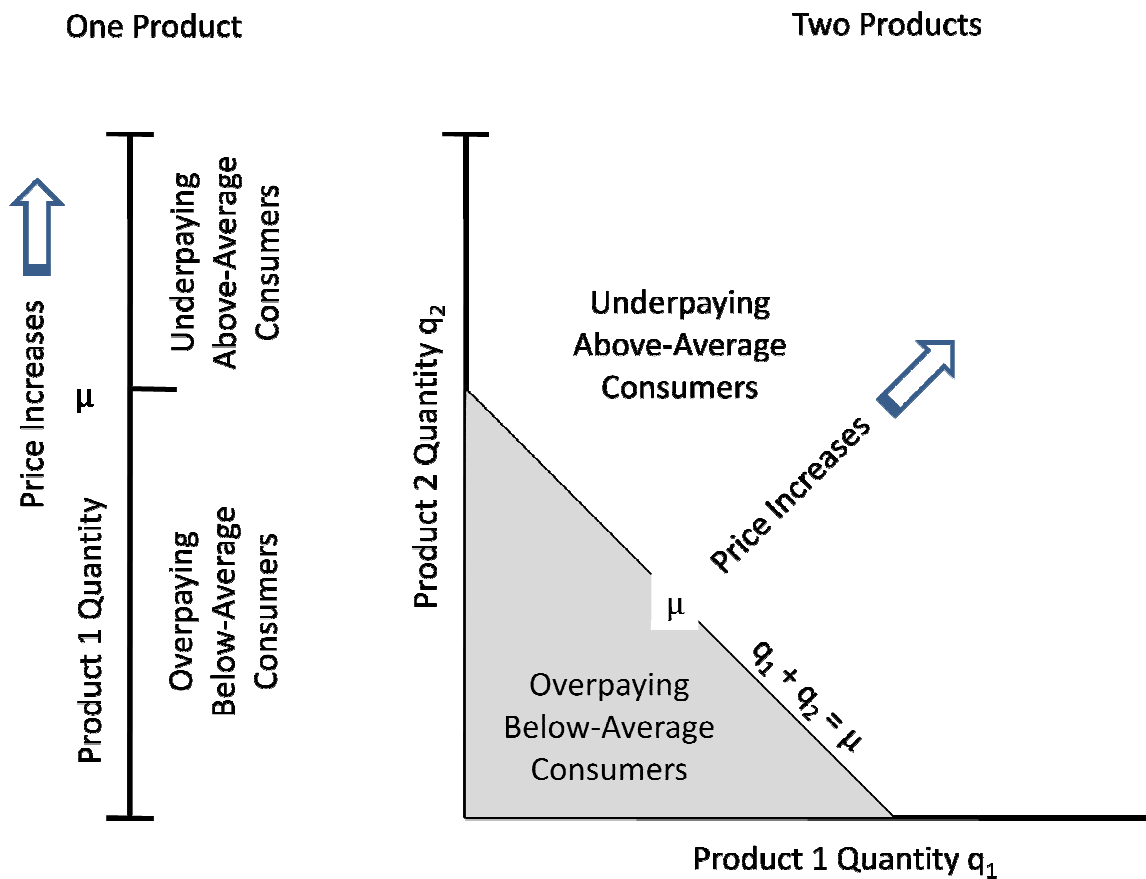
FIGURE 11: Multiproduct Customer Separating Effect

Since we can view two products as a single basket, the chart on the left applies to one product and two product consumers, since it shows average consumption. However, the chart on the right helps explain what is going on. To consider a simple example, suppose the two products are honeydews and canteloupes, and assume that each sells for a dollar per melon. Some consumers might prefer honeydews, and eat, say, seven honeydews and only three canteloupes; others might be the reverse; and some might split their consumption evenly. Since they are the same price, we can just count melons, and observe that if the average

consumption from a flat-rate provider is five melons then they will each appreciate the consumer surplus associated with dining at that provider, while the price lasts. It is easy to see that we can use an appropriate weighting function when the price of melons varies, arriving at a single price based on the average consumption in each step, but that the same phenomenon will happen. Of course, the "heaviest" consumer may not be one that is extremal in any single product (i.e., dimension), merely the one with the highest totalvalue of consumption. We can clearly translate any number of dimensions into a single one by weighted summation, i.e., $price_1 \times quantity_1 + price_2 \times quantity_2 + \cdots$, whereupon the results arrived at earlier apply.

We have assumed a fixed level of consumption, but suppose it is not? There are many different scenarios to explore, for example, one in which every customer's usage is fully memoryless, so that in every time period the demand for each user is just drawn from the underlying distribution. If that happens, then the price should merely exhibit random fluctuation as the sample mean more or less reflects the underlying customer population mean. The reason is this: at time $t = 0$, $\boldsymbol{A}$ and $\boldsymbol{B}$ will both have an initial set of customers. We can expect half to be in $\boldsymbol{A}$ and half to be in $\boldsymbol{B}$, and expect $\boldsymbol{P}_{i,B,0}$ to reflect the mean of the underlying statistical distribution of customer usages. Under the sequential defection rule or under the simultaneous defection rule or anything in between, one, or many, or some defections will occur both ways. If the consumption level of each of these customers is then completely reset, the members of $\boldsymbol{B}$ will have consumption values randomly drawn from the underlying distribution, and thus the same expected price, i.e., $E(\boldsymbol{P}_{i,B,0}) = E(\boldsymbol{P}_{i,B,1}) = \cdots E(\boldsymbol{P}_{i,B,t})$.

Depending on the underlying distribution, we may in many cases, but not all, expect a normal distribution of the sample mean μ over time. Note that, the price $\boldsymbol{P}_{i,B,t}$, while stationary, does not converge. A terminal state can be reached, but there is no bound on the number of steps to reach one. Briefly, it can be reached "by accident," when only one customer is in $\boldsymbol{B}$ and it is a heaviest user. As before, this can happen on the initial step. We have to assume that this "kills" the process, otherwise, on the next step, the usage for that customer will be "reset," possibly to a level lower than all other customer usage levels, causing a massive group defection to $\boldsymbol{B}$ (if the Simultaneous Defection Rule is in effect).

Another scenario might be one where consumption is a martingale, e.g., is one-dimensional Brownian motion, where the consumption for a given user at a given step is a random delta from the prior step. Here, we need to understand how big the delta may be relative to the range of consumption and the number of customers. If the change is small, then the average price won't be impacted very much, and most customers (assuming a non-pathological underlying distribution) that were lighter-than-average users will still be eligible to defect and will. For wide swings and few customers, the process may behave more like the prior example.

6. PRACTICAL CONSIDERATIONS FOR FLAT-RATE VS. USAGE-BASED PRICING

We have examined the dynamics of a market system using an *idealized* model of a duopoly with one flat-rate provider and one consumption-based provider under an iterative pricing *tâtonnement*. Normatively, a number of additional factors also may incent providers, customers, or markets to evolve to flat-rate pricing, consumption-based pricing, two-part tariffs, or other schemes.

Capital expenditures vs. operating expenses: For extremely capital intensive businesses with relatively minor operating expenses, since capital expenditures are sunk and fixed in the short-term, the marginal cost of usage is essentially zero. Therefore, typically in competitive markets, price approaches marginal cost. If capital investments are measured in billions and marginal operating expenses for a unit of production approach zero, a flat-rate may be advantageous, especially since such a plan is likely to reduce transaction costs and thus potentially increase firm profitability.

However, if a business has light capital requirements and fixed costs but heavy variable operating expenses, then a variable usage-based rate makes sense. As Nahata *et al* state, "in general, a fixed investment that reduces future variable production costs favors buffet pricing, but if it reduces future transaction costs, a two-part tariff or multipart tariff becomes more profitable."[lxxxvii]

Fixed costs vs. variable costs: Fixed costs are invariant in business volume. They may be tied to capital expenditures, e.g., depreciation, but may also represent non-volume sensitive overhead costs, e.g., the corporate jet. All other things being equal, the greater the ratio of variable costs to fixed, the more appropriate a usage-based plan may be.

Congestion and utilization: During periods of peak use, congestion of networks, or high resource utilization, marginal costs may no longer be zero as a next quantum of capacity needs to be added. Consequently, either usage-based or dynamic pricing must be used to disincent use during such periods, e.g., City of London congestion fees.

Discrete quanta of capacity increase: Part of the challenge is that when a service requires heavy capital investment, but resources are underutilized, the marginal cost of an additional unit of service may be near zero. However, when the resources are highly utilized, the marginal cost to service an additional unit of demand may be enormous. Consider an airline with one jet plane running a flight with empty seats on it. The marginal cost of an additional passenger is essentially zero. However, if the flight is full, the marginal cost is the cost of a new plane, or at least an entire flight. An entire field of accounting has been borne of this dilemma, e.g., Long Run Incremental Cost accounting, and a variety of proposals have been put forth to match pricing to marginal cost under

congestion, e.g., MacKie-Mason and Varian's smart markets. Flat-rate pricing is challenged to perform well in such environments.

Consumer and provider visibility into usage, billing transparency, auditability: For usage-based schemes to be effective, both consumers and providers must have visibility into real-time, current, and historic usage patterns, in terms of metered units as well as charging implications. After all, if a provider were just to say to a customer: "you owe me $27,000," it is unlikely that a customer would be willing to pay. Itemized bills and/or independently verifiable audit trails are essential. Of course, these add transaction costs which may shift the breakeven usage point and therefore self-selection outcome for any given consumer. For usage-based plans, providers typically must offer a verifiable chain of information to document and validate consumption in case of billing disputes. This ties into transparency via, e.g., itemized bills such as call detail records.

Measurement of delivered product / service quantity and quality: To what extent are customers assured that the quantity of melons that they receive is what they paid for, and that they are ripe as can be? Such quantity uncertainty and quality uncertainty are the domain of Akerlof's lemons markets and subsequent works. Similarly, for best-efforts statistically multiplexed services with overbooking, say, data networking, is the user getting as much bandwidth as they are paying for? Is it meeting network service level objectives and agreements regarding jitter, latency, and packet loss? In a cloud computing context, resources are typically provisioned on an abstract container basis, without exact guarantees as to, say, the number of operations that will be performed or equivalent measures to jitter, the number of operations that will be guaranteed in each millisecond.

Expectations of future usage: Regardless of historical usage and projected trends, customer *ex ante* expectations of future usage may vary from *ex post* results. Recall that Lambrecht and Skiera propose an "overestimation effect," where expectations of future consumption are higher than actual, driving a flat-rate bias.

Action-orientation and ability: Rational consumers may realize that given their historical usage and projected future usage, defecting to a different price plan or provider would maximize surplus. However, they may not have the time or inclination to incur cognitive, emotional, or physical transaction costs involved in the switch, or may not have the ability, due to contractual obligations or procedural constraints, possibly including non-tariff barriers.

Technology for metering and displaying usage. Usage may need to be monitored at an instantaneous peak or integrated over time. Today we view electricity metering as straightforward and as indivisible from electric service, but there was a time when electricity could only be measured in terms of

instantaneous use. It took the invention of the pendulum-based meter by Hermann Aron in 1883 and then the patented improvement of the electro-mechanical induction watt-hour meter by Elihu Thomson in 1888 to greatly improve on measurement[lxxxviii]. While metering may thus seem straightforward, it will be appreciated that other pay-per-use schemes may involve other technologies. For example, pay-as-you-drive (PAYD) requires telemetry regarding miles driven, and perhaps where they are driven (congested and/or unsafe roads), when, and how quickly. This in turn may involve global positioning system technology, wireless technology, and perhaps large-scale data analytics. Presumably very few customers stand outside their house watching the electric meter indicators spin. However, new smart-grid technology and computer-based tools are enabling exactly this, from a PC or smartphone. Such technologies may in turn impact consumption.

Timing of payment and compliance: One advantage of a flat-rate plan is that the total charge is known in advance of consumption, consequently, it is easy, although not a necessity, to charge people in advance of delivery of service. An amusement park, for example, charges an entry fee, rather than rendering a bill upon exiting the park. However, for consumption-based pricing, it typically is not feasible to render a bill in advance. It is hard to anticipate that you will talk to Aunt Martha for exactly twenty-seven minutes three weeks from now, and be presciently billed accordingly. Of course, exceptions exist in both cases: buffet restaurants often render a bill on exit, not entry, and pre-paid cell phones require payment in advance; whether you call Aunt Martha or not you have a fixed number of minutes of fungible capacity. If bills are rendered *ex-post*, there may be issues with payment, delays of payment, and uncollectible accounts. Also, payment timing and delays can introduce issues ranging from cash flow and cash management to time value of money to cognitive biases such as hyperbolic discounting.

Simplicity and customer comprehensibility: Flat-rate plans are simpler to calculate and easier for customers to comprehend than pay-per-use, which is easier than a two-part tariff, which is easier than a three-part tariff. Pay-per-use may seem just as simple, but consider how many people can accurately project whether $29.95 per month is more or less than $1 per gigabyte.

Costs of variable quantity delivery: There may be costs associated with the ability to deliver a variable quantity of goods or services. For example, in the case of electricity, the ability to vary usage depends on installing switches or potentiometers (e.g., dimmers), and in the case of variable delivery of water, faucets are required. This may seem obvious, but as Kolay and Shaffer[lxxxix] point out, in some domains, it may be cheaper to deliver predefined price-quantity bundles (i.e., various sizes of packages), than to put an apparatus such as a "dispensing machine" in each store capable of measuring exact quantities of say, potato chips. This in some cases argues against variable quantity whether under

flat-rate tariffs or usage-sensitive ones, and provides support instead for price-quantity bundles.

Cost of and process for metering usage: Technologies such as these aren't free. In the case of mobile networks and global positioning, investments may already have been made and therefore there is zero or minimal marginal cost associated with their use. However, there is still the expense of actual metering, e.g., building and maintaining the meter and/or telemetry capability. Some metering has an additional cost of operation, e.g., running a traditional electromechanical electric meter requires use of some of the electricity being metered, and this is a fraction of the electricity that is remotely generated given various losses in the system due to transmission and distribution. Information technologies such as wireless, microprocessors, and policy-based agents are reducing these costs, however.

MacKie-Mason and Varian point out that an issue with usage-sensitive pricing is accounting and billing costs. As an extreme case, they point out that overhead costs would be "astronomical" if it were necessary to provide detailed accounting on every IP packet traversing the Internet.[xc] They observe that a variety of strategies can be used to minimize the load, e.g., statistical sampling, accounting at a fine-grained basis only during periods of congestion, and use of a distributed architecture for message accounting data. While they suggested statistical sampling, most individuals probably think of water meters, electric meters, and item-level restaurant bills when they think of usage-based accounting, all arguably based on direct measurement, not sampling. After all, the server does not evaluate the food on the table at periodic intervals, he or she accounts for the two club sandwiches and the salad at time of ordering. However, statistical sampling is in fact often used in usage-based schemes. For example, some compute resources offered under usage-sensitive prices are billed based on CPU utilization; said utilization measured by a software agent every 15 minutes and averaged over an entire month to render a bill.

Time limits: When consumption is correlated to time, a time limit can serve to act as a consumption limit, improving the economics of flat-rate plans. Nahata *et al*[xci] posit that this is why some restaurants offer lunch buffets, where consumption may be constrained by limited duration of lunch breaks, whereas they don't offer dinner buffets, where presumably much more could be consumed. Another effect they posit is that transaction overhead due to usage-based schemes could limit customer transaction throughput, thus constraining revenue.

Intrinsic limits of consumption: Satiation can prevent infinite consumption of some goods. Nahata *et al* point out that diners do have some upper limit on consumption, which they denote by q*. However, it is useful to distinguish between a customer-driven limit of consumption, and other bottlenecks. For example, an all-you-can-eat buffet customer may not consume all the food that

they've brought to the table, but this does not mean that the restaurant does not incur the cost of purchasing the food. Similarly, a customer may not "consume," i.e., watch, all of the episodes and movies that they have recorded to a personal video recorder, but this does not mean that an IPTV provider does not incur capital costs for network build-outs and upgrades and possible congestion during peak periods as the customer downloads or streams all of that content.

Validity of technology and models to forecast usage: Health insurance initially may have been a domain of information symmetry via mutual ignorance. However, underwriting models based on large-scale data analysis may have favored the insurer, whereas information asymmetries ("I smoke 2 packs a day, but I'm not putting that down on my application") may have favored the insured. Technologies such as DNA analysis and increased identification of correlation between genetic markers and predisposition to diseases can help either insurers or insured parties or both to better forecast future usage and therefore impact adverse selection or consumption quantity uncertainty effects.

Other transaction costs: In addition to costs of metering, there may be many more transaction costs associated with usage-based models, including rating (calculating bills based on raw usage, non-linear volume discounts, taxes, bundles, promotions, and the like), rendering a bill (delivering an electronic or physical copy of a bill), accounts receivable, collections, account management, billing dispute management, etc. These have different domain-specific characteristics. A restaurant offering *à la carte* pricing may need to train or otherwise familiarize servers with pricing and pricing changes, print menus, adjust prices, reprint menus, have point-of-sale terminals with customized entries (hamburger with or without cheese, with or without onions, with or without mayo, etc.). Attributes of each model also generate hidden costs and externalities: a buffet restaurant must dedicate floor space and refrigeration to the bar, but an *à la carte* restaurant is a make-to-order custom shop rather than a make-to-stock, which creates its own issues with inventory management and order processing. *À la carte* often requires wait staff, whereas in a buffet there is a hidden cost to the customer of self-service. Automats replace the operating expense of wait staff and bus boys with the capital expense of vending machines. *Kaiten sushi*, or conveyer-belt sushi, replaces wait staff with a conveyer belt and a manual accounting system with different color plates.

These transaction costs are non-trivial. Nahata *et al* argue that transaction costs associated with usage-based pricing can be so substantial as to make flat-rate pricing more profitable in real life[xcii]. Sundararajan[xciii] remarks that for many information goods, where the marginal cost of production and distribution is essentially zero, it is these transaction costs that can dominate. For example, in selling one digital song for .99 cents, the marginal cost to "reproduce" and distribute is trivial. However, suppose that the customer then has an issue with billing and spends an hour on the phone with a customer service representative, costing ten dollars?

It has been suggested by Coase[xciv], that firms exist to minimize transaction costs, which to extend his classification scheme somewhat, perhaps may be classified into categories such as search, information, bargaining and negotiation, execution, i.e., physical performance of the transaction, monitoring, control, coordination, and contract maintenance and enforcement, all of which help sway the decision between flat-rate and usage-based.

***Sharing*:** Sometimes sharing is explicitly prohibited and sometimes not. Sharing constraints are artifacts of flat-rate pricing and may also impact costs. For example, broadband access and all-you-can-eat buffets are precluded from being shared either by contract, rule, or norm. Social norms intermingle with formal rules: one is not supposed to pass food acquired under buffet pricing to another diner at the table paying on an *à la carte* basis. On the other hand, in less developed countries, electricity and/or water may be shared by all inhabitants of an apartment block due to the cost of metering. The ability to share under flat-rate plans appears to vary with context. Consumer home internet services were initially unshareable even between computers in the same home owned by the same family: one PC equalled one modem equalled one dial-up line equalled one account. Wi-Fi reduced technical constraints on sharing. As wireless networks began to become prevalent, one could ostensibly tap into a neighbors unsecured access point with no apparent harm to the neighbor.   However, normally, a customer is not permitted to share "unlimited" broadband Internet access with a neighbor under a single flat-rate fee. However, sharing of the connection is certainly permitted among family members. However, some wireless broadband plans do not permit tethering, that is, letting a single user with multiple devices share a connection and subscriber identity among devices. These are all artifacts of flat-rate pricing, and linear usage-based plans clearly are not susceptible to these issues. Under usage-based plans, sharing might actually be encouraged, since it saves the provider the cost of laying access to the customer that is piggybacking on the service relationship, while retaining the revenue from that customer. In others, such as restaurants, there are still scarce resources whose cost must be recovered, e.g., tables and seats, so there may be a sharing charge even under usage-based plans.

Resaleability and transferability: Acquiring goods under flat-rate plans could be extremely profitable for a customer if "sharing" via resale and/or title transfer were permitted.   Various technologies and domains create their own issues here. Information goods without digital rights management are an example.

***Costs for splitting, combining, or arbitrage*:** Depending on specific industry context, costs of multiplexers, inverse multiplexers, "break bulk," transport, and transformation provide economic and technological constraints in addition to social or regulatory limits on resale.

Perishability and Transportability: While resaleability and transferability address legal and contractual constraints, perishability and transportability address physical realities. If melons can be purchased for a volume-insensitive flat fee, instead of a cupful of melon slices, why not walk out of the restaurant with millions of dollars in melon inventory and then resell it? Perishability, physical inability to transport, or nontrivial losses in transport can act as barriers when legal constraints don't.

Income effects and price elasticity of demand: Altering behavior such as consumption requires that the economic decision be perceived as worthwhile and that there be some price elasticity of demand. If charges are a trivial portion of personal or household income, price effects may not drive consumption. And, for some items there may be little or no price elasticity of demand.

Substitution effects: substitution effects reduce demand for a good by replacing it with a different one. For example, a substitute for a particular pay-per-use television show may be a different one, going to a movie, or even going out to eat or meditating. User willingness to consume a substitute may depend on pricing plans for both and also marginal cost under either plan.

Relationships, billing frequency and bill size, intermediaries: Electricity is pay-per-use, but every single electron consumed does not trigger a billing event. Transaction costs for usage-based schemes are partly dependent on frequency of bill generation and whether relationships exist. Odlyzko[xcv], in assessing the viability of micropayments, points out that "accounted systems, such as [metered electricity], which keep track of tiny transactions and bill for them at the end of a period" are subject to different overheads than micropayments. He points out that ringtone and song downloads are subject to similar characteristics. Intermediaries can reduce these costs, e.g., credit cards can reduce some transaction costs by aggregating billing events, and buying a variety of products from a single retailers or etailer can as well. Consumer perception is important, smaller bills more often vs. larger less often can have a variety of impacts: immediate ability to detect and respond to changes, and indifference to charges. For example, many people would rather pay four dollars for gourmet coffee every weekday of their working years than be presented an equivalent bill for $50,000 for the right to do the same (50 weeks * 5 days / week * $4/day * fifty years (say, an age of twenty to seventy).

Incentive compatibility: Tariffs that incent people to act in accordance with and thereby reveal their true needs are considered incentive compatible. For example, signing up for a flat-rate text messaging plan that has a breakeven point at 50 text messages presumably reveals that one intends to send at least 50 text messages. Such tariffs are incentive compatible if customers then truthfully reveal their needs or preferences, and ideally drive desirable behavior. For example, pay-as-you-drive insurance programs which are priced based on

miles driven have the desirable characteristic of increasing the price of driving, thus reducing demand, and reducing carbon emissions and congestion.

Active and visible vs. passive and hidden use: Automation enables much more passive and hidden use. For example, an electric customer is surely aware when he actively turns a light switch on or off, but may not be aware of the power draw of a cable set top box that is always on even if the TV is off. An internet user is presumably aware when they choose to download a file (although some files, such as automatic updates may not have such characteristics), but may not be aware of increasingly passive applications, e.g., peer-to-peer file sharing applications and always-on nanny cams. These applications are expected to consume an increasing amount of Internet bandwidth.[xcvi] Lack of visibility and control can cause a number of issues such as fear and uncertainty which all feed into loss aversion biases.

Customer control of consumption, automated policy management: Similar to light switches, as a rule, customers in restaurants can control what they order, avoiding, e.g., the "market priced" Surf & Turf special of lobster tails and filet mignon in favor of the Chicken Caesar salad. But sometimes, when hosting a dinner, the guests may order the most expensive bottle of wine on the menu, so control is not assured. Another example of lack of control is when the payor is not the user, as anyone with a teenager (who texts) is well aware. One means of addressing this issue is automated policy management, e.g., systems that only allow 10 text messages per day for a teenager.

Predictable expenditures: clearly better for flat rate plans.

Congestion and frustration: Imagine being stuck in traffic. Now imagine being stuck in traffic while a taxi meter is running. Equivalently, imagine a slow internet connection. Now imagine such a connection when you are paying by the minute for access, as under older dial-up plans. Price plans may impact user satisfaction with services which may impact churn which impacts profitability.

Contract and billing periods, term and termination: Mechanisms have arisen to enable a reduction in the variability of pay-per use bills given customers' loss aversion, dislike of uncertainty, and preference for predictability. Two that come to mind are averaging using forecasted use based on historic seasonality, e.g., of natural gas (used for heating) in colder climates, and the use of "rollover minutes." Both help smooth billing swings to a lower coefficient of variation than the underlying usage variation. Contract periods reduce churn and therefore overhead costs, and also increase switching costs for customers thus reducing defections.

Services may be offered on a no-commitment basis, e.g., taxi cabs or cloud computing virtual servers, or under a short-term, e.g.,daily or monthly or longer-term, e.g., 3 year (network services), 6, 7 or 10 year (outsourcing) contracts, or

even 99-year (Hong Kong concession) lease basis. Some contracts may be terminated early, but either only under certain conditions (non-performance, malfeasance), or with the payment of fees. Such fees may reflect residual value, e.g., for a smartphone in a cellular service contract or a car under a lease, but appear as switching costs.

Overheads and chaos: Overhead coupled with economies of scale can create chaos, i.e., extreme sensitivity to initial conditions. Initial random allocations of customers to providers due to notable early contract wins, say, may enable one provider in a multi-provider context to benefit from a winner-take-all effect. In this paper, we have ignored overheads, but surely in some markets at some times there are first mover advantages, just as in others at other times there are fast-follower advantages.

SLAs: In the real world, it is virtually impossible for even the best service providers to deliver "perfect" service 100% of the time. Issues such as floods, fires, outages, pandemics, acts of god, component reliability issues, etc., impact service delivery quality. Consequently, service level agreements can be important. Service level agreements require careful capacity planning, engineering, and management, such management in turn requiring performance fencing, private or virtual private resources, and/or congestion control.

Lock-in and switching costs: Often there are costs associated with defecting. These include costs to the customer in switching providers, and costs to the provider in switching customers. These may include contracts for customer premises equipment, knowledge of procedures, uncertainties regarding practices and quality of the new provider ("the devil you know vs. the one you don't"), termination and disposition fees, initiation or application fees and so forth. A correct economic decision requires a risk-adjusted, discounted cash flow perspective to determine whether the likely discounted savings are greater than the total switching costs, but bounded rationality suggests that such decision-making processes will be rarely executed in practice. From a holistic viewpoint, in addition to hard dollar costs, other cognitive, emotional, or physical costs may need to be taken into account as well. Switching costs may also be indirect. For example, frequent flyer plans, frequent (hotel) stay plans, or other frequent buyer plans induce customers to limit the selection of providers so as to build up credits with a select few.

As an example of relatively high switching costs, consider a large global company who is desirous of switching their IT infrastructure, currently owned and therefore loosely equivalent to a flat-rate, to a cloud computing infrastructure. Such an endeavor arguably might require security studies, rewriting applications potentially with tens of millions of lines of code, retraining developers, redesigning enterprise applications architecture, and the like. On the other hand, one example of minimal switching costs is the U. Cal. Berkeley INDEX study,

where switching between flat-rate and usage-based plans was designed to be as simple as a single mouse click[xcvii].

Zone of indifference and satisficing: Switching costs are not only hard dollar, but may include cognitive, emotional, or physical costs. Cognitive costs include the intellectual effort to determine the optimal rational (expected utility, risk-adjusted, discounted cash flow) choice, subject to biases, heuristics, and bounded rationality. Emotional costs may include enduring contract renegotiation. Physical costs may include the effort of walking to a store to switch providers, or driving further to get a better deal. Nobelist Herbert Simon is known for drawing the distinction between ideal neoclassical maximizers and more real-world satisficers, who select not the optimal solution, but a good enough one.

Indifference has the following effect on re-pricing. First of all, the larger the level of indifference, the sooner the terminal state is reached. In the limit, of course, if everyone was completely indifferent, the initial state would be the terminal state: no one would have the impetus to defect. In general though, there are two factors at work. First, we are "done" more quickly…some transitions will never have to be conducted. For example, if indifference is at 25%, once μ is within 25% of c_{max}, any users in the top 50% of consumption (assuming uniform distribution) will not be able to defect. The other reason is that price increments tend to be larger, as those with consumption near the average μ won't be compelled to defect.

Credence goods, service inducement and moral hazard: We normally think of consumption levels as being determined by the customer. However, for some goods the level of consumption is, if not decided, then "induced" by the provider. Examples of such "service inducement"[xcviii] or "supplier-induced demand"[xcix] are when an auto mechanic suggests a new transmission, a doctor recommends a follow-up visit, a management consultant proposes additional analysis of a market opportunity, or a cab driver selects a route to the destination. Debo *et al*[c] observe that these are particularly likely for "credence" goods, whose quality cannot be determined even after consumption, as opposed to "experience" goods, whose quality can be determined by the customer during consumption. Information asymmetry and a type of moral hazard are at work here: only the "expert" knows what is truly required. Under flat-rate charges, the expert is incented to avoid additional work ("no, I think it's fine, really"), whereas under usage-based charges, the expert is incented to suggest additional work.

Capacity constraints: As we have discussed, capacity constraints may be a cause or result of various pricing schemes: they may drive congestion fee or usage-based pricing, or may be a result of flat-rate pricing.

Entry strategy, customer self-awareness, market maturity: Flat rate may be better to enter a market, when users are unsure of their consumption levels. On

the other hand, pay-per-use may be better, as one may want a free or inexpensive taste before committing to a sizeable recurring charge.

Choice vs. churn: The relative importance of factors in choosing a provider may not be identical to those in customer retention, the flip side of which is churn, or defection. For example, in one study,[ci] word of mouth recommendation, reliability, and speed were top reasons for selecting an Internet Service Provider, but price was the number one for defecting (others included service, support, and billing issues). The key learning is that numerous factors are present in the decisions to select, continue with, or defect from a provider. However, in support of the arguments herein, customers certainly do consider price and are willing to take action to defect.

Implementable and scalable: per packet and dynamic pricing have been proposed for the Internet, where each packet determines whether to be sent based on instantaneous spot prices given the value to the application / customer of it's being sent, but the mechanics of implementation are viewed as potentially creating too much overhead. Other schemes may be "somewhat" implementable, but not scale efficiently.

Perfect information: A key assumption underlying defection is the notion that a customer has information concerning the price of both providers. However, as Stiglitz has convincingly argued[cii], real markets don't necessarily provide such information, and there are a number of paradoxes regarding using information to make decisions, e.g., suppose that the cost of acquiring the information is equal to or greater than the benefit from applying the information in decision-making? One reason information is important is that "it enables excess price differences to be dissipated [and] the allocation of goods across markets to be efficient."[ciii]

Information technology and informationalization: If patronage and defection decisions as well as transaction costs determine ultimate viability of pricing plan options, the impact of information technology cannot be denied. For example, transaction costs for measured service based on distance for subways have been reduced in many cities via RFID cards that can exactly determine origin-destination pairs and thus usage-based (perhaps proxied by distance-based) plans. Even the capital expense of the RFID chips can be offset by refundable customer deposits for the cards. In an increasing number of countries, road-use meters, also known as smart meters[civ], can identify distance driven and location to provide consumption-based charging. As the cost of technology—wireless, mobile, GPS, sensor networks, etc.—continues to decrease, transaction costs approach zero, in turn tipping the balance increasingly to pay-per-use.

Economies of scale: In the analysis above, we have assumed that each provider can acquire products (melons) at identical cost m, and sell at an identical unit price M or at an equivalent average selling price (for flat-rate providers with immediate re-pricing). Instead of attempting to recapitulate entire

textbooks of material on competitive strategy and economics, we simply note that there may be factors such as overhead costs, stair-step effects in capital expenditures, buyer power over suppliers and the like that favor larger providers. During an early stage of market ecosystem evolution, large providers with economies of scale certainly may offer flat-rate, utility plans, two-part tariffs, multipart tariffs, or some or all of the above. Consequently, a new entrant may be at a disadvantage. If the usage-based providers have the natural advantage discussed here, and in an early stage of system evolution they already have scale advantages, then they may be expected to continue to preferentially attract non-heavy customers, and in addition, to even attract heaviest customers. Suppose it is the flat-rate provider that begins with the scale advantage? Well, if there is sufficient spread in consumption between the lightest user and the heaviest user, even with a unit price disadvantage a new pay-per-use entrant may acquire customers from a larger flat-rate provider with scale economies. The market system will then continue to evolve to a point where a "somewhat" heavy user is no better off defecting to a utility provider. Of course, depending on the lifetime of capital assets deployed and numerous other factors, at this point the utility provider may have achieved scale economies over the flat-rate provider.

Statistics of scale: The term "economies of scale" traditionally relates to production economies. However, there is another effect that I've christened the "statistics of scale,[cv]" which relates to benefits of scale associated with beneficial statistical multiplexing of demand. Briefly, when demand is aggregated from multiple customers there is an aggregate demand smoothing effect, and a measure of variability, the *coefficient of variation*, tends to decrease. To the extent that flat-rate providers can reduce transaction friction and thus handle greater volume, this effect may provide incremental advantage to the flat-rate provider.

Ability to re-price: In the dynamic described in this paper, it is assumed that the one or more flat-rate providers re-price at each time period. However, regulation may limit the allowable frequency of re-pricing actions, or require that no customer is "worse off" after such re-pricing.

Legal considerations: Although beyond the scope of this paper, legal considerations such as the Robinson-Patman Act may impact flexibility relative to the full range of theoretical pricing options.

Behavioral economics and cognitive biases: The number of cognitive biases applicable to provider selection, price-plan selection, and defection could fill its own volume. We will only touch upon some of the highlights here.

Elsewhere, I have described these dimensions of human behavior as "Lazy, Hazy, Crazy."[cvi] People can be "lazy," in that while economists may sometimes only consider quantifiable hard dollar costs, people also consider cognitive,

emotional, physical, and opportunity costs. For example, how complex is the math to forecast usage and determine which plan is best? How much of an emotional toll is there in arguing with the customer service representative over a billing error? How physically challenging or time-consuming is it to drive over to another provider to review competitive offers when Billy's soccer game starts in only an hour. They are often "hazy," that is, use heuristics and approximations to make decisions. And, they are often "crazy," in the sense of making decisions not just on purely "rational" factors.

Loss aversion: Perhaps most important is Kahneman and Tversky's Loss Aversion[cvii], which posits asymmetric effects in consumer perception of gains and losses. Simply put, the pleasure associated with gaining a dollar is less than the pain perceived by losing one. This has important ramifications in the discussion of flat-rate vs. usage-based plans, as from a behavioral economics perspective, the key benefit of flat-rate plans is that the user avoids the pain associated with potentially unbounded loss from consumption or billing gone awry. This is one of the drivers of the "flat-rate bias."

Choice-supportive bias: Switching costs such as contract termination fees were discussed earlier. To this should be added the "choice-supportive" bias, in which decision-makers rationalize earlier decisions, ignoring data that would disconfirm the decision made. This creates friction in defection, because these biases can detract from the rational decision-making presumed herein.

Framing: People make decisions based on how choices are framed, as any advertiser knows. Most customers would rather "avoid the hassle of complex bills," but on the other hand, "why overpay?"

Fairness: The "Ultimatum Game"[cviii] has two players. One determines how to split a windfall amount, and the other can accept the split whereupon both players get their share. However, if the second player refuses the split, neither player receives any payoff. Humans have a strong sense of fairness[cix]: in virtually all cultures (all except one), players offered substantially less than 50-50 splits will refuse them. This is not rational, since any amount (even under a 99.99 to .01 split is better than none.

The interaction of fairness, framing, and flat-rate vs. usage-based pricing presents some interesting challenges. After all, one might argue that a flat-rate school tax is fair, since the entire community benefits from educated children, or that it is fairer for only parents of school-age children to pay for education.

Too many choices: More choices, e.g., of rate plans and providers, are often viewed as beneficial for consumers as well as social welfare. However, research has shown[cx] that too many choices can actually reduce consumption and purchase levels, presumably due to the added cognitive overhead of engaging in complex decision-making processes.

Zero-price effect: Shampanier and Ariely[cxi] offered customers a choice between a free ten-dollar gift certificate and a twenty-dollar gift certificate priced at seven dollars. Although the latter choice offered higher economic value, not just a majority, but all of the customers selected the free option. This has positive and negative implications for flat-rate plans. We already know that since the marginal cost to a customer of consuming an additional item is zero, consumption is likely to be higher than if there were a positive price for the item. One may conjecture that Ariely's zero-price effect suggests that actual consumption might even be higher than otherwise projected.

Social norms vs. pure rationality: Interestingly, Shampanier and Ariely also report decreased consumption in some circumstances when price is reduced to zero. For example, when people were offered candy at one cent per piece, market norms took precedence, and recipients bought multiple pieces. However, when the price was reduced to "free," social norms took precedence, and participants only took one piece.

Size of compelling differences: If you were about to buy a new car and the salesperson said "I shouldn't tell you this, but the same car is in stock across the street for $12,000 less", you'd probably run across the street. However, if the same car were in stock across the street and available for 12 cents less, not many people would bother. This is not just due to the economic value of the time to cross the street: behavioral studies have shown that the brain's decision-making apparatus is an imperfect analog calculator, and it is not the absolute difference, but percentages and how questions are framed that can impact decisions. In prior sections, we have assumed that decision-makers are rational and will switch.

Given all of these different factors, provider determination of profit-maximizing plans and customer behavior when faced with a variety of provider plans can be non-trivial to predict.

7. SUMMARY

There are a large number of pricing plans in use across a variety of industry segments, each with its own advantages and disadvantages. Two key ones across many sectors are certainly flat-rate and usage-based. Flat-rate plans have a number of benefits, such as simplicity, previsibility, mitigation of consumer loss aversion issues, and a reduction in transaction costs. As our idealized model shows, however, in a duopoly the flat-rate plan may be unsustainable and is thus dominated by usage-based pricing, and usage-based pricing has many benefits including eliminating moral hazard and adverse selection effects.

We have also argued for a shift in perspective on information asymmetry, quality uncertainty, moral hazard, and adverse selection. Briefly, information asymmetry may not be the cause of some market failures previously ascribed to it, which may instead be due to sufficiently dispersed heterogeneous consumption under flat-rate pricing. The notion of "quality uncertainty" does not precisely capture analogous effects due to consumption quantity uncertainty under such consumption dispersion, although there is a synthesis of both effects which may be simply described as rational agents only conducting transactions at actual value, rather than expected, when actual value is known by at least one party in advance. Moral hazard, we argue, has little to do with morality, but a lot to do with rational indifference to level of consumption at zero marginal cost. Adverse selection, we argue, is nothing more than rational surplus-maximizing consumers actively self-selecting optimal providers or plans.

We have used an idealized model to determine the system dynamics of defections in a multi-price-plan ecosystem using formal analysis and computer simulation. However, it is clear that the real world is much more multidimensional. For example, Webber[cxii] explores in detail the introduction by The Dialog Corporation, an online information provider, of a new pricing mechanism which moved away from connect-time charges and more towards flat fees. Issues of trust, fear of surprises, and the like were rampant in a negative customer reaction to the shift. Conversely, Webber contrasts Dialog with ESA-IRS, which also shifted price plans but positioned this shift as a shared learning experience, resulting in greater customer satisfaction. A body of research (for example, see Kolay and Shaffer [cxiii]) exists examining optimal strategies and conditions for the success and profitability of various plans, which is beyond the scope of this paper. In all, these results may be summarized as "it depends." However, based on our model and some arguments, some observations may be made.

If demand is identical across all customers and time periods, either flat-rate pricing or usage-based pricing is sustainable and the charges are identical under either plan.

If demand is dispersed but completely stochastic, then not only are there no salient information asymmetries, but paying or receiving a payment in an amount equal to the expected value of the transaction, e.g., flat-rate pricing, is rational and sustainable.

It is when demand is dispersed but predictable that challenges arise for the sustainability of flat-rate pricing. These fundamentally have less to do with information asymmetry than the interplay of dispersed consumption with flat rates. As stated above, when value is dispersed, no rational agent will choose to transact an exchange at average value if such an exchange would lead to a loss. This means that in lemons markets, rational sellers will not sell quality cars at an expected value, and that in melons markets, rational buyers will not pay flat rates

if they are light consumers. Information asymmetries may affect the dynamics of quality variation, but are less relevant to quantity variation *unless the provider can choose not to serve some subset of customers*, in a way related to the mirror-image case where a buyer can choose not to buy a given car from a given seller or subset of sellers. To understand why information asymmetries are less relevant in the case of quantity variation, imagine an all-you-can-eat buffet where customers come in with an exactly truthful planned consumption quantity visible on their name badge, with a matching certificate of planned consumption signed and notarized and easily visible to the buffet manager, and moreover that their forecast planned consumption was 100% accurate. Light eaters would still choose to go across the street to the regular (usage-based pricing) restaurant, once they realized that they were paying as much as the heaviest eaters at the buffet and could save money by defecting to a usage-based plan. If a provider could screen by asking "how much do you plan to eat" before deciding *whether* to serve the customer, then asymmetries, truthfulness, and incentive compatibility play a role. This is not that farfetched: insurance companies ask "how much do you plan to eat" by determining pre-existing conditions, ensure truthfulness under threat of refusing to pay out on claims, and create incentive compatibility via a menu of pricing and deductible options.

The monotonically increasing price under defections from flat-rate plans has an interesting dynamic, explored at length in the paper. Not examined, but worth mentioning, is that there is likely to be a similar, but inverted, curve describing the dynamics of the devolution of market function described by Akerlof, where the average price monotonically decreases as average quality continuously decreases in lemons markets.

This paper began simply as an observation that defection of light users under flat-rate plans would inevitably lead to price increases in turn causing more defections, and turned into a longer examination of the dynamics of the effect. It is my hope that researchers, academics, and professional economists and mathematicians will be able to conduct a much more thorough investigation or comment on my arguments if they deem them of interest.

8. APPENDIX: HTML/JAVASCRIPT SIMULATION

Use a text editor to save the code below as a file "sim.htm", open sim.htm in a Javascript-enabled browser, cut and paste the output and save as a file "results.csv", then open in a spreadsheet program for charting / analysis.

```
<html>
<head>
<title>Market Simulation</title>
<script type="text/javascript" language="javascript">
   var n = 100 /* number of customers - set to any value but sim runs in time O(n squared) */
   var spaces = ",    "
   var timestep = 0
   var isflatrate = new Array(n)
   var candefect = new Array(n)
   var consumption = new Array(n)
   var total, flatcount, flatprice, candefectcount, i, selected

   function simulate()
      {
      document.write("t" + spaces + "Flat-Rate P" + spaces + "|A|" + spaces + "|D|<br //>")
      for(i=0; i<n; i++)
         {
         consumption[i] = i + 1 /* for uniform distributions, change to "= Math.random() * n" */
         isflatrate[i] = (Math.random() < .5)
         }
      calcandprint()
      while(candefectcount > 0)
         {
         selected = Math.floor(Math.random() * (candefectcount - .000001))
         i = 0
         while(selected > 0)
            {
            if(candefect[i]) selected -= 1
            i++
            }
         while(!candefect[i]) i++
         isflatrate[i] = !isflatrate[i]
         calcandprint()
         }
      }
   function calcandprint()
      {
      timestep += 1
      total = flatcount = candefectcount = 0
      for(i=0; i<n; i++)
         {
         if(isflatrate[i]) total += consumption[i]
         if(isflatrate[i]) flatcount += 1
         }
      flatprice = total / flatcount
      for(i=0; i<n; i++)
         {
         candefect[i] = (isflatrate[i]) ? (consumption[i] < flatprice) : (consumption[i] > flatprice)
         if(candefect[i]) candefectcount += 1
         }
      document.write(timestep + spaces)
      document.write(flatprice.toFixed(6) + spaces)
      document.write((n - flatcount) + spaces + candefectcount + "<br //>")
      }
</script>
</head>
<body onload="simulate();" style="font-family:Courier,monospace;"> </body>
</html>
```

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